3450:436/536 Mathematical Models, Kreider, Spring 2014

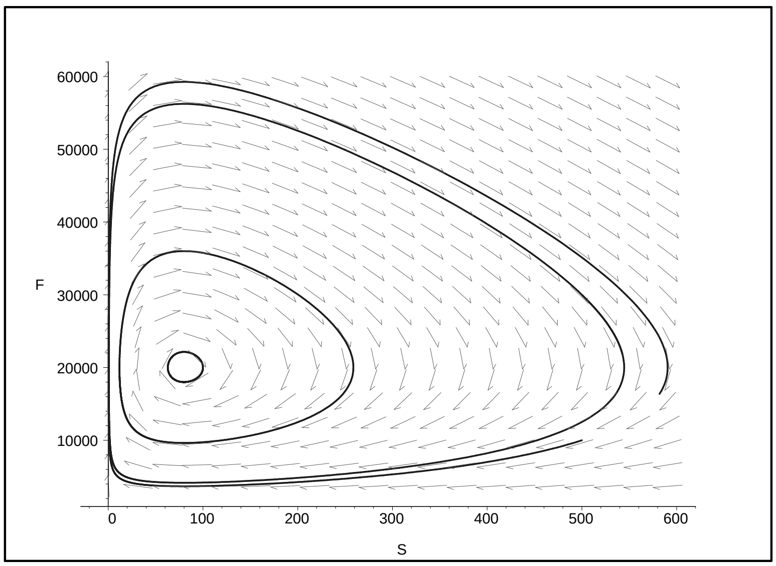
Plots for the predator prey model

The predator prey model, with $F(t)$ as the fish (prey) population and $S(t)$ as the shark (predator) population, is

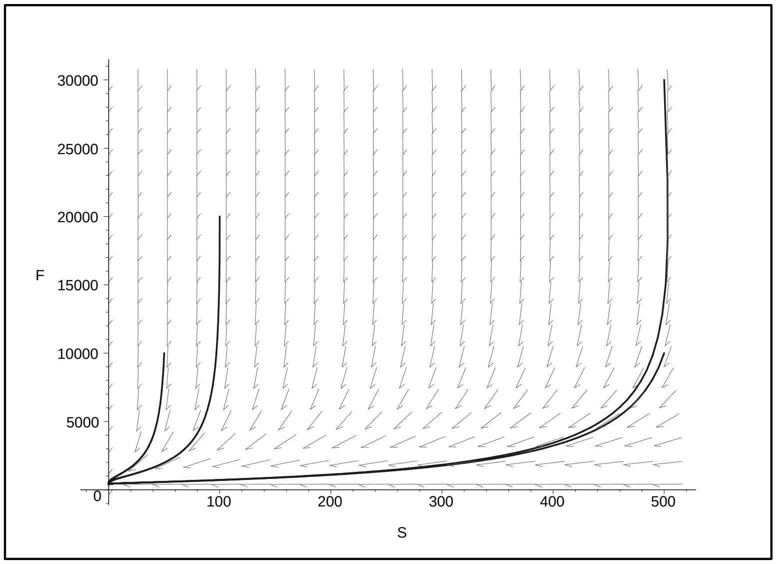
$$\begin{aligned} F' &= aF - bF^2 - cFS \\ S' &= -kS + \lambda FS \end{aligned}$$

Baseline values are $a = 0.04$, $b = 10^{-4}$, $c = 0.0005$, $\lambda = 10^{-5}$.

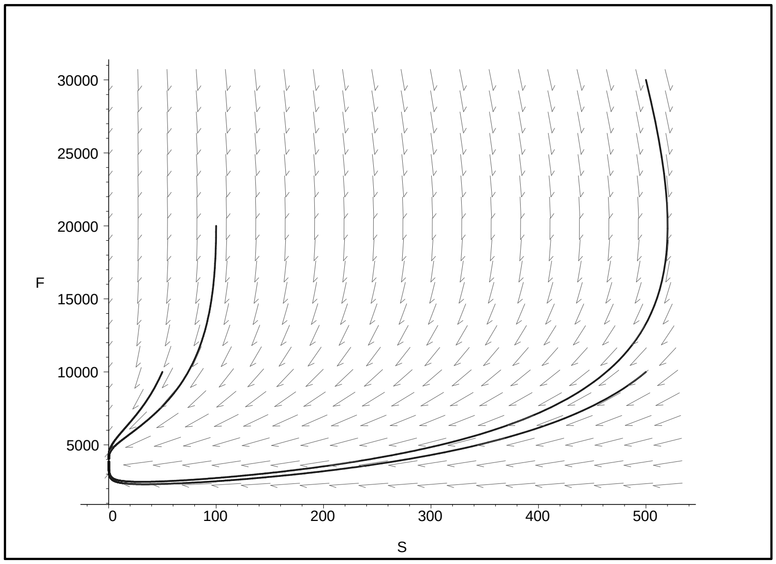
Exponential Growth. If b is set to 0, then the fish population grows exponentially. The critical point $(a/c, k/\lambda) = (20000, 80)$ is a center. Here are a few trajectories.



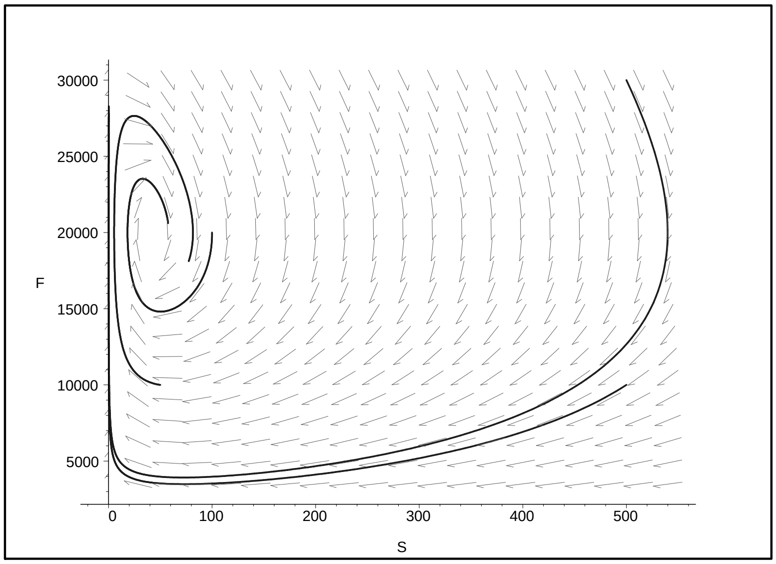
Logistic Growth with $b = 10^{-4}$. For this large logistic term, the equilibrium shark population $S_{eq} = (a - bk/\lambda)/c$ is negative. This implies that there cannot be a stable shark population – the large logistic term means the fish population cannot rebound quickly enough from predation. Once the sharks are extinct, the fish equilibrium population is $a/b = 400$.



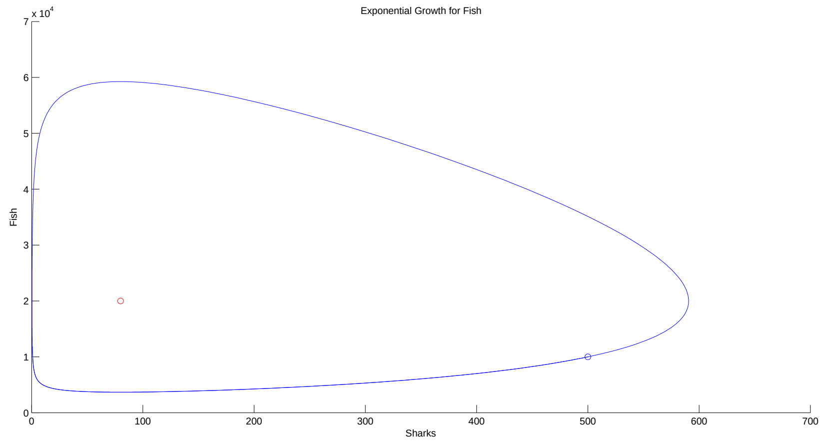
Logistic Growth with $b = 10^{-5}$. For this large logistic term, the equilibrium shark population $S_{eq} = (a - bk/\lambda)/c$ is negative. This implies that there cannot be a stable shark population – the large logistic term means the fish population cannot rebound quickly enough from predation. Once the sharks are extinct, the fish equilibrium population is $a/b = 4000$.

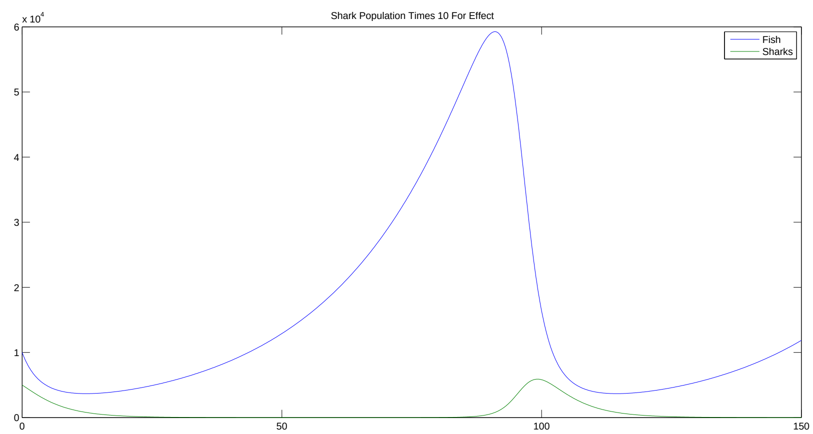


Logistic Growth with $b = 10^{-6}$. For this small logistic term, the equilibrium shark population $S_{eq} = (a - bk/\lambda)/c$ is now positive (40). The equilibrium point is stable spiral.

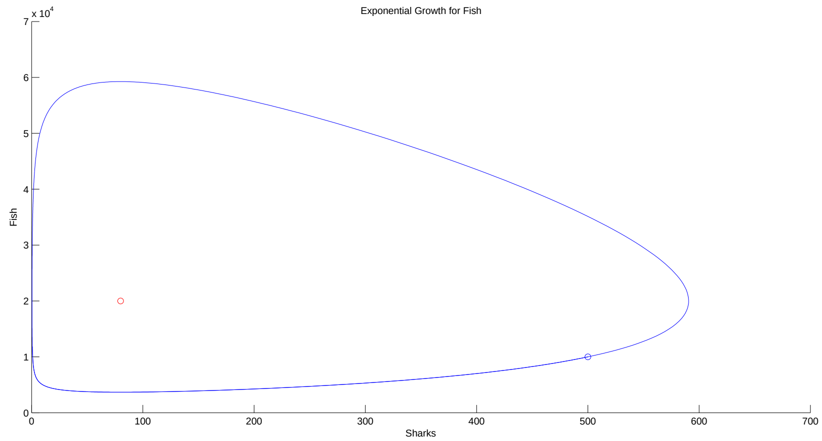


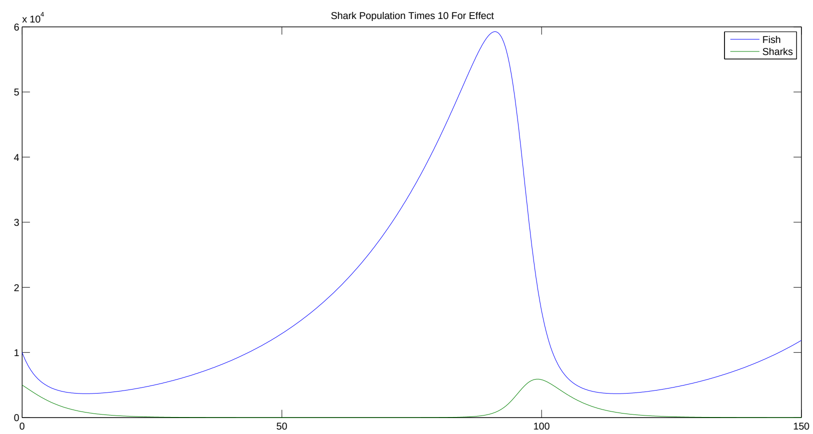
Exponential Growth. Here is a comparison of the phase portrait and the cartesian plots for $F(t)$ and $S(t)$ when b is set to 0.



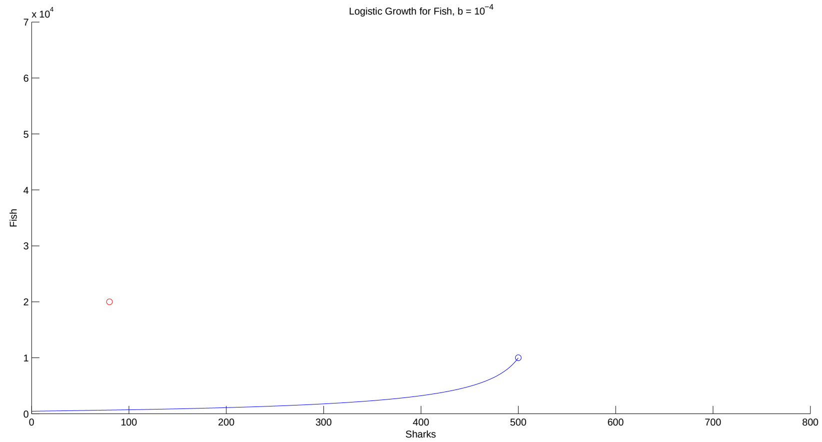


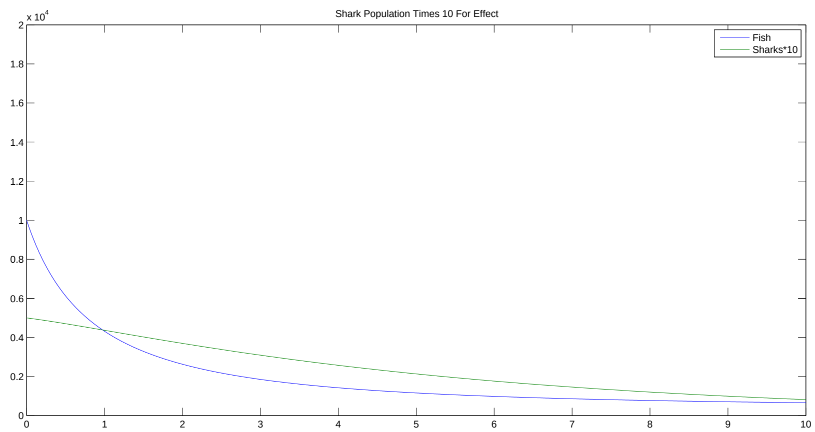
Logistic Growth. Here is a comparison of the phase portrait and the cartesian plots for $F(t)$ and $S(t)$ when $b = 10^{-4}$.





Logistic Growth. Here is a comparison of the phase portrait and the cartesian plots for $F(t)$ and $S(t)$ when $b = 10^{-5}$.





Logistic Growth. Here is a comparison of the phase portrait and the cartesian plots for $F(t)$ and $S(t)$ when $b = 10^{-6}$.