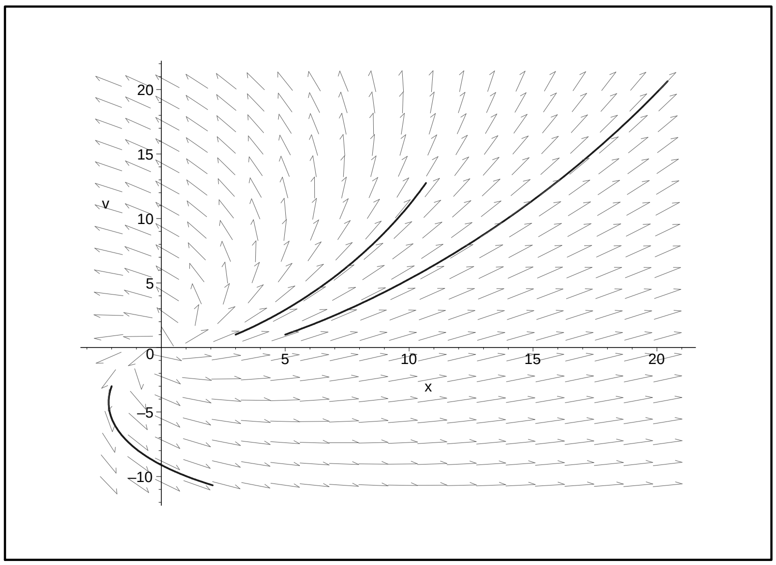
3450:436/536 Mathematical Models, Kreider, Spring 2014

Phase Portraits

The direction field and a few trajectories near the critical point (0,0) for the system

$$\begin{aligned} x'(t) &= 2x(t) - v(t) \\ v'(t) &= x(t) + v(t) \end{aligned}$$

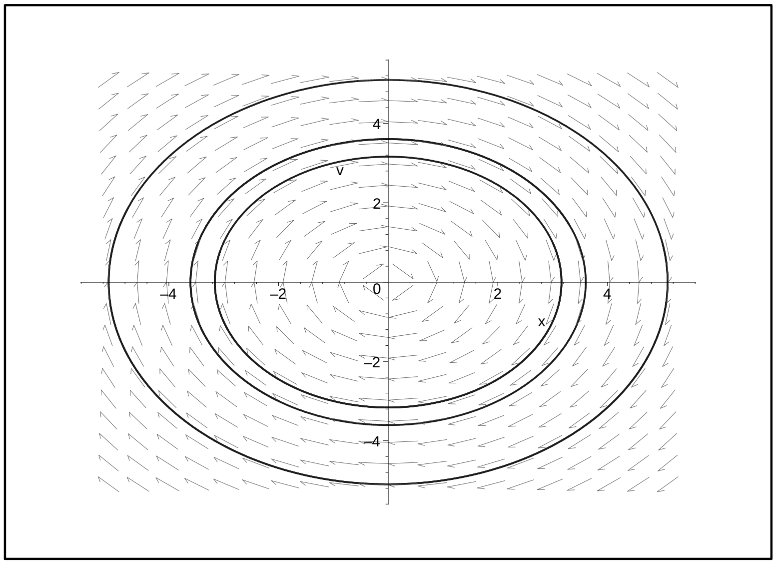
are shown below. The critical point is an unstable spiral.



For simple harmonic motion, given by

$$\begin{aligned} x'(t) &= v(t) \\ v'(t) &= -x(t) \end{aligned}$$

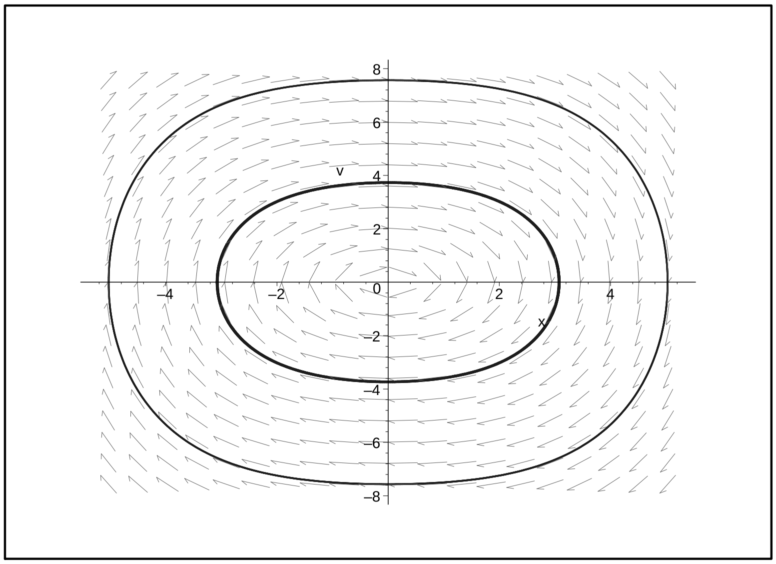
the trajectories are circles. The critical point is a center.



For the hard spring with no damping, given by

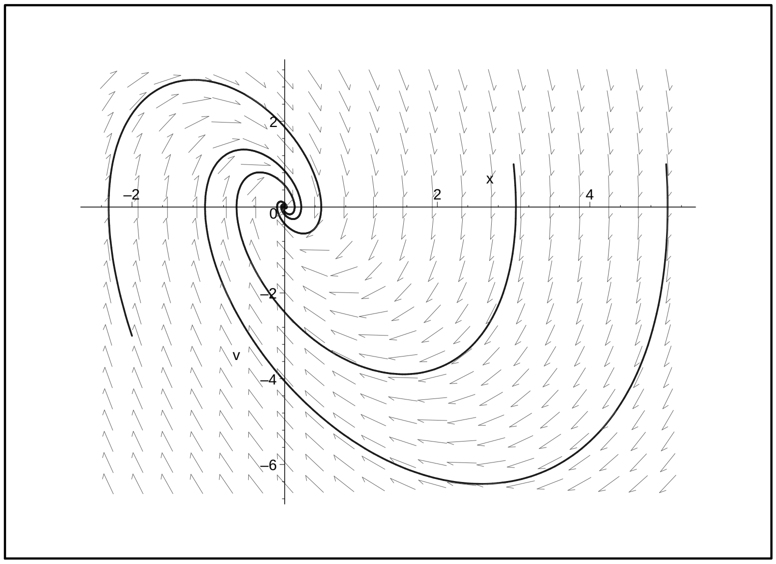
$$\begin{aligned} x'(t) &= v(t) \\ v'(t) &= -x(t) - 0.1x^3(t) \end{aligned}$$

the trajectories are flattened, and indicate a higher velocity than the corresponding linear spring above. The additional restoring force leads to higher velocities, and faster oscillations (note that the phase diagram doesn't give you a good sense of time). The critical point is a center.



For a linear spring with damping, the critical point is a stable focus. Here,

$$\begin{aligned} x'(t) &= v(t) \\ v'(t) &= -5x(t) - 2v(t) \end{aligned}$$



The nonlinear system

$$\begin{aligned} x'(t) &= v(t) \\ v'(t) &= 1 - \cos(x(t)) \end{aligned}$$

has a more complicated phase diagram.