3450:436/536 Mathematical Models, Kreider, Spring 2014

Homework Set 7

Due date: Wed 30 April

Type your responses to the extent possible. If necessary, leave blank space in the document to write equations by hand.

1. (30 pts) A hedge fund employs financial managers at 3 levels: Junior Managers, Senior Managers and Partners. Over the course of a year,

   - a Junior Manager has a 5% chance of being promoted to Senior Manager, a 1% chance of leaving the firm, and a 6% chance of going to jail for professional misconduct;
   - a Senior Manager has a 3% chance of being promoted to Partner, a 2% chance of leaving the firm, and an 8% chance of going to jail for professional misconduct;
   - a Partner has a 3% chance of leaving the firm and a 2% chance of going to jail for professional misconduct;
   - anyone in jail has an 80% chance of being paroled and returning to the firm as a Junior Manager.

   (a) Viewing this as a discrete Markov process for the states

   $\{\text{Junior Manager}, \text{Senior Manager}, \text{Partner}, \text{Jailed}, \text{Left the Firm}\}$, write the transition matrix P.

   (b) Compute the probability that a Junior Manager will be in jail during the third year (apply 2 year-long stages of the Markov process). You may compute this by hand or use the matrix.

2. (30 pts) Consider a system with n active components and one spare, with failure rate λ failures/year. Modify the reliability model derived in class to find the analytic expression for the reliability $R(t)$ of the system if failure of the system is defined as the failure of 2 or more of the individual components. First write this in general terms using n and λ, and then write the explicit expression for a system where $n = 7$ and $\lambda = .01$ failures/year.