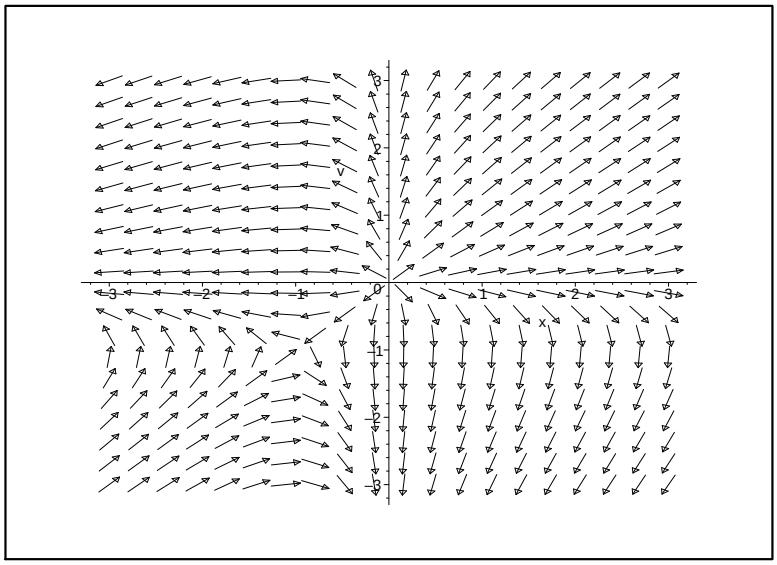
3450:436/536 Mathematical Models, Kreider, Spring 2014

Homework Set 6

Due date: Wednesday 9 April

Type your responses to the extent possible. If necessary, leave blank space in the document to write equations by hand.

1. (20 pts) Here is a phase plane diagram. Locate each critical point by drawing a dot on the plot. Classify the critical point (center, saddle, node, spiral, stable or unstable, etc). Sketch by hand the trajectories that start at $(-0.1, -0.5)$ and $(0, 1)$.



2. (20 pts) Consider the damped pendulum equation, $\theta'' + \epsilon\theta' + \sin\theta = 0$. Write this as a 2×2 first order system using variables θ and $v = \theta'$. Perform linear stability analysis on the system and identify the eigenvalues $\lambda^{\pm}$. Use the eigenvalues to identify the 2 different types of behavior that could occur in the system, based on the value of ϵ. How do you distinguish the different behaviors?

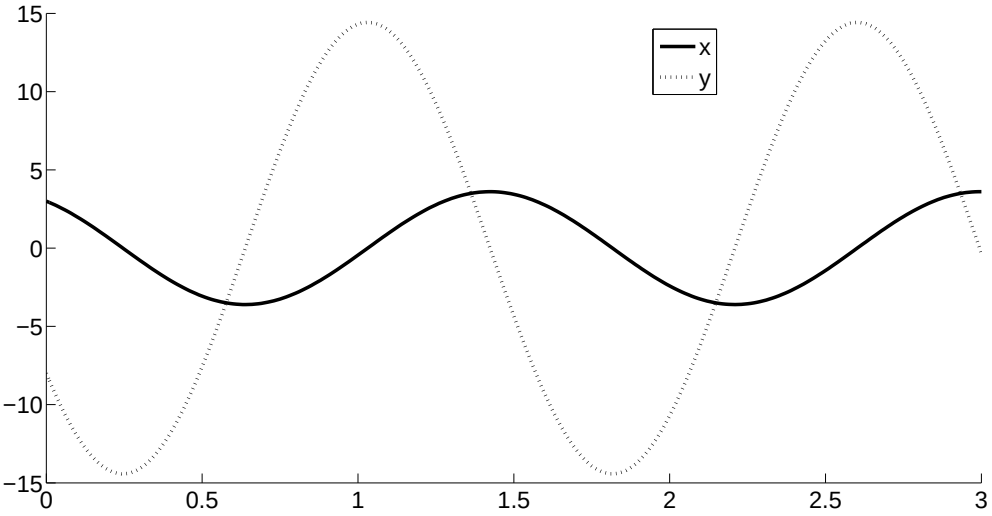
3. (20 pts) Consider the predator-prey model with no logistic term:

$$\begin{aligned} F' &= aF - cFS \\ S' &= -kS + \lambda FS \end{aligned}$$

   Show that the nonzero critical point is a center, implying periodic behavior in the populations.

4. (20 pts) Given the cartesian plots of $x(t)$ and $v(t)$ on the back, draw a rough sketch of the corresponding trajectory in the phase plane. Use an arrow to indicate the direction of travel.

5. (20 pts) Consider $y' = r + y^2$ with $r < 0$. Use linear stability analysis to show that the base state $\bar{y} = -\sqrt{-r}$ is stable.



6. (30 pts) Find the bifurcation diagram for $y' = ry - y^2$.

7. (20 pts) 536 STUDENTS ONLY. Here is a phase plane diagram. Sketch the trajectory that starts at $(0, 0.25)$. Describe in words what is happening to the object represented by the system, assuming that the horizontal axis x is the position and the vertical axis v is the velocity.