Homework Set 6

Due date: Wednesday 9 April

Type your responses to the extent possible. If necessary, leave blank space in the document to write equations by hand.

1. (20 pts) Here is a phase plane diagram. Locate each critical point by drawing a dot on the plot. Classify the critical point (center, saddle, node, spiral, stable or unstable, etc). Sketch by hand the trajectories that start at \((-0.1, -0.5)\) and \((0, 1)\).

2. (20 pts) Consider the damped pendulum equation, \(\theta'' + c\theta' + \sin \theta = 0\). Write this as a 2 \(\times\) 2 first order system using variables \(\theta\) and \(v = \theta'\). Perform linear stability analysis on the system and identify the eigenvalues \(\lambda^\pm\). Use the eigenvalues to identify the 2 different types of behavior that could occur in the system, based on the value of \(\epsilon\). How do you distinguish the different behaviors?

3. (20 pts) Consider the predator-prey model with no logistic term:

\[
    \begin{align*}
    F' &= aF - cFS \\
    S' &= -kS + \lambda FS
    \end{align*}
\]

Show that the nonzero critical point is a center, implying periodic behavior in the populations.

4. (20 pts) Given the cartesian plots of \(x(t)\) and \(v(t)\) on the back, draw a rough sketch of the corresponding trajectory in the phase plane. Use an arrow to indicate the direction of travel.

5. (20 pts) Consider \(y' = r + y^2\) with \(r < 0\). Use linear stability analysis to show that the base state \(\bar{y} = -\sqrt{-r}\) is stable.
6. (30 pts) Find the bifurcation diagram for \( y' = ry - y^2 \).

7. (20 pts) 536 STUDENTS ONLY. Here is a phase plane diagram. Sketch the trajectory that starts at \((0, 0.25)\). Describe in words what is happening to the object represented by the system, assuming that the horizontal axis \( x \) is the position and the vertical axis \( v \) is the velocity.