3450:436/536 Mathematical Models, Kreider, Spring 2014

Homework Set 4

Due date: Wednesday 5 March

Type your responses to the extent possible. If necessary, leave blank space in the document to write equations by hand.

1. (30 pts) Write the model equations for the following predator-prey system, involving caribou and tundra wolves in Canada. This is a modification of the standard model presented in class. Use whatever symbols you want for the proportionality constants, but write a short paragraph that defines your terminology.

   Caribou features

   - Standard natural birth and logistic terms for population growth
   - Standard "loss due to predation" interaction term
   - Additional natural death term, where the death rate is proportional to population

   Tundra wolf features

   - Standard "growth due to predation" interaction term
   - Standard natural death term, where the death rate is proportional to population
   - Additional death term due to hunting by humans, proportional to the square root of population

2. (20 pts) Write the rate equations for the chemical reaction

$$Fe^{2+} + 2OH^{-} \rightleftharpoons Fe(OH)_2$$

   with forward reaction rate k^+ and backward reaction rate k^-. Use your choice of notation for the concentrations.

3. 536 STUDENTS ONLY. (20 pts) Consider the system of 2 lakes (labelled A and B), 3 rivers (labelled 1,2 and 3) and 2 factories (labelled F_1 and F_2) shown here. Consider the mass of some type of pollutant that flows through the system. Denote the total mass, in kg, of the pollutant in Lake A as $P_A(t)$, and that of Lake B as $P_B(t)$. Write the system of ODEs for these 2 quantities using the idea of conservation of mass, and the notation below.

   The conservation of mass for the mass of pollutant in one lake has the format

$$\frac{dP}{dt} = \text{input rate} - \text{output rate}$$

   where the units are kg/time (let's use days as the time unit). Note that there may be several inputs and/or outputs for any given lake; those terms are summed to give the net effect.

   Input rates may be either direct (dumping from a factory), with units kg/days, or calculated from an input flow (inflow from a river). In this case, the input rate is the pollutant concentration of the inflow (kg/ m^3) times the flow rate (m^3/day). For the output rate, assume that the pollutant is well-mixed in the lake (no spatial variation), so the concentration in the lake is $P(t)/V$, where V is the constant volume. To maintain the constant volume assumption, assume that the flow rates of all 3 rivers are the same, $r = 10^5$ m^3/day, and ignore evaporation, rainfall and similar phenomena.

Denote the pollutant concentration in River 1 as $c_1 = 0.02$ kg/m^3, the volumes of the lakes as $V_A = 10^8$ m^3 and $V_B = 10^6$ m^3, and the amount of dumping from the factories as $F_1 = 10^3$ kg/day and $F_2 = 5 \times 10^2$ kg/day.