3450:436/536 Mathematical Models, Kreider

Review of ODE techniques

Wikipedia has reasonable descriptions of all of these techniques.

1. Integrating Factors. For an equation of the form $y' + P(t)y = Q(t)$ (first order linear), define $k(t) = exp\Big(-\int P(t)dt\Big)$. Multiply the equation by k to get $(ky)' = kQ$ and integrate both sides. Don't forget to add the constant of integration.

2. Bernoulli equations. This is a special form $y' + P(t)y = Q(t)y^n$, where n is not 0 or 1. Make the substitution $u = y^{1-n}$, so that $u' = (1-n)y^{-n}y'$, replace the y' in the equation with the expression involving u' and simplify. The result will be an integrating factor problem. Solve it for u, then convert back to y.

3. Underdetermined Coefficients. This is for second order linear equations: $ay'' + by' + cy = f(t)$, where f is either a polynomial, a sine or cosine, an exponential, or a sum and/or product of those forms. Find the homogeneous solution $y_h = c_1y_1 + c_2y_2$, and then build the form of the particular solution based on the structure of f. If there is duplication with the fundamental solutions, multiply the form by the smallest power of t that eliminates the duplication. Substitute that form into the ODE to get equations for the unknown parameters.

4. Variation of Parameters. This is for second order linear equations: $ay''+by'+cy = f(t)$, where f is anything. Find the homogeneous solution $y_h = c_1y_1 + c_2y_2$ and build the Wronskian W. The particular solution has the form $y_p = u_1y_1 + u_2y_2$ where $u_1' = -fy_1/W$, $u_2' = +fy_1/W$.