Wikipedia has reasonable descriptions of all of these techniques.

1. Integrating Factors. For an equation of the form \( y' + P(t)y = Q(t) \) (first order linear), define
   \[ k(t) = \exp\left(-\int P(t)dt\right). \]
   Multiply the equation by \( k \) to get \((ky)' = kQ\) and integrate both sides. Don’t forget to add the constant of integration.

2. Bernoulli equations. This is a special form \( y' + P(t)y = Q(t)y^n \), where \( n \) is not 0 or 1. Make the substitution \( u = y^{1-n} \), so that \( u' = (1-n)y^{-n}y' \), replace the \( y' \) in the equation with the expression involving \( u' \) and simplify. The result will be an integrating factor problem. Solve it for \( u \), then convert back to \( y \).

3. Underdetermined Coefficients. This is for second order linear equations: \( ay'' + by' + cy = f(t) \), where \( f \) is either a polynomial, a sine or cosine, an exponential, or a sum and/or product of those forms. Find the homogeneous solution \( y_h = c_1y_1 + c_2y_2 \), and then build the form of the particular solution based on the structure of \( f \). If there is duplication with the fundamental solutions, multiply the form by the smallest power of \( t \) that eliminates the duplication. Substitute that form into the ODE to get equations for the unknown parameters.

4. Variation of Parameters. This is for second order linear equations: \( ay'' + by' + cy = f(t) \), where \( f \) is anything. Find the homogeneous solution \( y_h = c_1y_1 + c_2y_2 \) and build the Wronskian \( W \). The particular solution has the form \( y_p = u_1y_1 + u_2y_2 \) where
   \[ u_1' = -f y_1 / W, \]
   \[ u_2' = +f y_1 / W. \]