3450:221, Calculus I, Review for Exam 1 Key

1.

2. Start with $|f(x) - L| < \epsilon$: here, it is $\left|6 - \frac{4}{7}x - 50\right| < .3$. This simplifies to $\left|\frac{-4}{7}x - \frac{8}{7}\right| < .3$. Factor out the $-4/7$ to get $\frac{4}{7}|x+2| < .3$ so that $|x+2| < .3 * 7/4 = .525$. Use $\delta = .525$.

3. a) Factor to $x-2$; $L = -5$. b) Common denominator up top gives $1/3x$; $L = -1/9$. c) Conjugate gives $1/(\sqrt{x+1}+1)$; $L = 1/2$. d) Direct substitution gives $L = 0$. e) No cancellation: L dne. f) Simplify to $3+x$; $L = 3$.

4. a) $\lim_{x\to 2^+} \frac{x-2}{x-2} = 1$. b) $\lim_{x\to 2^-} \frac{-(x-2)}{x-2} = -1$. c) dne.

5. Listed in text. Be prepared to write these and use them in standard problems on the exam.

6. Let $f(x) = x^3 - x^2 + x - 20$. $f(2) = -14$ and $f(3) = 1$. Since f is continuous (it's a polynomial), the Intermediate Value Thm applies and hence there must be some point c where $2 < c < 3$ for which $f(c) = 0$.

7. f has a jump at $x = 0$ and an infinite discontinuity at $x = 2$. Note that the asymptote is only on the right, but that's enough for us to call it an inf. disco.

8. The rate of change of C is $C'(x) = 1.5 - 500/x^2$. When $x = 500$, $C'(500) = 1.5 - 1/500$.

9. The difference quotient is $\frac{1}{h}\left(\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right) = \frac{-1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$. Taking the limit gives $\frac{-1}{(x)(2\sqrt{x})} = -\frac{1}{2}x^{-3/2}$. At $x = 4$, the slope of the tangent is $-1/16$, so the line is $y - 1/2 = -1/16(x-4)$.

10. a) 2. b) dne; jump at $x = 4$. c) 7. d) 0 (it's the slope at $x = 2/5$). e) 2 (slope at $x = 1$).

11. a) False; there might be a removable discontinuity. b) False; this just says it works for *one* ϵ; it must work for *all* ϵ for the limit to exist. c) Might be true or false. If $x = a$ is not in the domain, there might just be a hole (the limit exists), or there might be a jump or a vertical asymptote (the limit dne).

12. a) $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$. b) $G = \frac{\sqrt{g}-17}{g^2}$, where $g(t) = t^2 - 2$. $f(x) = \frac{\sqrt{x}-17}{x^2}$.

13.

14.

15.

16. $f'(x) = \frac{3}{4}x^{1/3} - \frac{27}{64}x^{-7/4}$. $g'(x) = \frac{[2x+1](x^2+1) - [2x](x^2+x+1)}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$.

$h'(x) = \left[-4x^{-5} - \frac{1}{11}x^{-2/11} - 35x^{-10/3}\right]\left(\sqrt{x} - \frac{x}{x+1}\right) +$
$\left(x^{-4} - \frac{1}{9}x^{9/11} + 15x^{-7/3} + 12\right)\left[\frac{1}{2}x^{-1/2} - \frac{1}{(x+1)^2}\right]$.