3450:221, Calculus I, Review for Exam 1

Assuming the exam questions will be identical to the homework and sample exam questions could be hazardous to your grade. To study for the exam, you should review the sections, class notes, assigned homework problems and quizzes. Above all, **work out problems**, don't just glance at previously worked problems.

1. Read the handout Being Prepared For Calculus on my web site.

2. Use the $\epsilon - \delta$ definition of the limit for $\lim_{x\to -2}(6 - 4x/7) = 50/7$ to find δ if $\epsilon = 0.3$.

3. Find each of the following limits, if it exists. If it doesn't, explain why not. You need not list the limit laws you used, but do show any algebraic manipulation.

   a) $\lim_{x\to -3} \dfrac{x^2 + x - 6}{x + 3}$ b) $\lim_{x\to -3} \dfrac{1/x + 1/3}{x + 3}$

   c) $\lim_{x\to 0} \dfrac{\sqrt{x+1} - 1}{x}$ d) $\lim_{x\to 8} \dfrac{x - 8}{\sqrt[3]{-2}}$

   e) $\lim_{x\to -1} \dfrac{x^2 + x - 2}{x^2 - x - 2}$ f) $\lim_{x\to 0} \dfrac{(1 + x)(2 + x) - 2}{x}$

4. Consider the function $f(x) = \dfrac{|x - 2|}{x - 2}$. Find each of the following limits, if it exists. If it doesn't, explain why not. a) $\lim_{x\to 2^+} f(x)$ b) $\lim_{x\to 2^-} f(x)$ c) $\lim_{x\to 2} f(x)$

5. Statement of definitions and theorems. a) Define "f is continuous at $x = 1$". b) Define "$\lim_{x\to a} f(x) = L$". c) State the Intermediate Value Theorem. d) Define "f is differentiable at $x = a$". e) State the Squeeze Theorem.

6. Prove that there is a solution of the equation $x^3 - x^2 + x = 20$ in the interval $[2, 3]$. Identify the theorem you are using and verify that it applies to this situation.

7. Find and classify the discontinuities of the function below.

$$f(x) = \begin{cases} 2x - 4, & \text{if } x \le 0; \\ x^2 - 3, & \text{if } 0 < x \le 2; \\ \frac{1}{x-2}, & \text{if } 2 < x \end{cases}$$

8. Trasho Manufacturing Inc figures that to make x plastic wastebaskets each week costs $C(x) = 100 + 1.5x + 500/x$ dollars. Calculate the "rate of change of C with respect to x" if 500 wastebaskets are produced weekly.

9. Use the limit definition of the derivative to calculate $f'(4)$ for $f(x) = \dfrac{1}{\sqrt{x}}$. Then write the equation of the tangent line to $y = f(x)$ at $(4, 1/2)$.

10. Refer to the graph of $y = f(x)$ below to answer the following questions:

    a) Find $\lim_{x \to 4^+} f(x)$, if it exists. If not, why not?

    b) Find $\lim_{x \to 4} f(x)$, if it exists. If not, why not?

    c) What value(s) for $f(6)$ should be used to make f continuous at $x = 6$?

    d) What is the instantaneous rate of change of f with respect to x at $x = 2.5$?

    e) Find $f'(1)$.

11. True or false. If a statement is false, either explain why it is false or give an example showing why it is false.

    a) If $\lim_{x \to a} f(x)$ exists, then $f(x)$ is continuous at $x = a$.

    b) If there is an $\epsilon > 0$ and a $\delta > 0$ so that $|f(x) - L| < \epsilon$ whenever $0 < |x - a| < \delta$, then $\lim_{x \to a} f(x)$ exists.

    c) If $x = a$ is not in the domain of $f(x)$, then $\lim_{x \to a} f(x)$ does not exist.

12. Let $G(t) = \dfrac{\sqrt{t^2 - 2} - 17}{t^4 - 4t^2 + 4}$. a) Find the domain of G. b) Write G as a composition of simpler functions: $G(t) = f \circ g(t)$. Write $f(t)$ and $g(t)$ explicitly.

13. Sketch the graph of a continuous function that is not differentiable at $x = 7$.

14. Sketch the graph of a function with the properties: $\lim_{x \to 1} f(x) = 3$, $f(1) = 2$, f is continuous everywhere except possibly at $x = 1$.

15. Sketch the graph of a function that is continuous everywhere except possibly at $x = 1$ and that has these properties: $\lim_{x \to 1^+} f(x) = 2$, $\lim_{x \to 1^-} f(x) = 3$, $f'(x) > 0$ for $x > 1$.

16. Differentiate these functions: $f(x) = \dfrac{9}{16}x^{4/3} + \dfrac{9}{16}x^{-3/4}$. $g(x) = \dfrac{x^2 + x + 1}{x^2 + 1}$.
    $h(x) = \left(x^{-4} - \dfrac{1}{9}x^{9/11} + 15x^{-7/3} + 12\right)\left(\sqrt{x} - \dfrac{x}{x+1}\right)$.