21. (5 pts) Find $\frac{dy}{dx}$ for $2x^4 + 3y^2 - y = x^2 + 2$

$$\frac{dy}{dx} = \frac{2x - 8x^3}{6y - 1}$$

22. (5 pts) Find $\frac{dy}{dx}$ for $1 + y = \sin(y^2x)$

$$\frac{dy}{dx} = \frac{y^2 \cos(y^2x)}{1 - 2xy \cos(y^2x)}$$
23. (5 pts) John stands on the shore of a river watching his house float downstream, parallel to the shore 50 ft from shore, at the rate of 25 ft/sec. How fast is the rate of change of the distance between John and his house changing? First write the general formula for the rate of change and then evaluate the formula when the house is 100 ft downstream from his position.

\[ D(t) = x(t) + 50^2 \]

\[ 2D \frac{dD}{dt} = 2x \frac{dx}{dt} + 0 \]

When the house is 100 ft downstream:

\[ D = \sqrt{x^2 + 50^2} \]

\[ \frac{dD}{dt} = \frac{x \frac{dx}{dt}}{\sqrt{12500}} \]

\[ x = 100 \]

\[ \frac{dx}{dt} = 25 \]

\[ D = \sqrt{100^2 + 50^2} = \sqrt{12500} \]

\[ \frac{dD}{dt} = \frac{100 \times 25}{\sqrt{12500}} = 22.36 \text{ ft/sec} \]

24. (5 pts) The brain weight of fish is modeled by the function \( B = .007W^{2/3} \), with \( B \), the brain weight, and \( W \), the total body weight, measured in grams. The body weight depends on the length \( L \), measured in cm, according to \( W = .12L^{2.53} \). If the average length of a species of fish, the Spiny Kreeiderian Hogfish, evolved from 15 cm to 20 cm over 1 million years at a constant rate, how fast was the brain growing at the instant when the average length was 17 cm?

\[ \frac{dB}{dt} = \frac{dB}{dW} \frac{dW}{dL} \frac{dL}{dt} \]

\[ = \left[ .007 \times \frac{2}{3} w^{-1/3} \right] \left[ .12 \times 2.53 L^{2.53} \right] \left[ 5 \times 10^{-6} \right] \]

\[ = \frac{1 \times 10^{-6} \text{ g}}{\text{yr}} \]

\( W = 155.17 \)

\( \frac{dL}{dt} = \frac{23.17}{1000000} \)

\( \frac{dW}{dt} = 3.17 \)

\( \frac{dL}{dt} = \frac{5 \times 10^{-6}}{} \)
25. (5 pts) Use a linear approximation to estimate \( \sqrt[3]{264} \):

\[
\sqrt[3]{x} \approx f(27) + f'(27)(x - 27)
\]

\[
\sqrt[3]{264} \approx 3 + \frac{1}{3 \cdot 27} (-1)
\]

\[
= 2.934
\]

26. (5 pts) The radius of a soccer ball is measured as 10.23 ± 0.4 in. What is the calculated volume? Use differentials to estimate the maximum possible error in the calculated volume. What is the relative error of the calculation?

\[
r = 10.23 \quad \text{and} \quad \Delta r = 0.4
\]

\[
V = \frac{4}{3} \pi r^3 \quad \Rightarrow \quad V = \frac{4}{3} \pi (10.23)^3 = 4484.5
\]

\[
\Delta V = 4 \pi r^2 \Delta r
\]

\[
|\Delta V| \leq 4 \pi (10.23)^2 \cdot 0.4 = 524.04
\]

\[
\frac{|\Delta V|}{V} \leq \frac{524.04}{4484.5} = 0.1173
\]

\[
\approx 11.77\%
\]
27. (5 pts) Find the critical points of \( f(x) = x^{4/5}(x-1)^{2/5} \)

\[
f'(x) = \frac{4}{5}x^{-1/5}(x-1)^{2/5} + \frac{2}{5}x^{4/5}(x-1)^{-3/5}
\]

\[
f'(x) = \frac{\frac{4}{5}(x-1) + \frac{2}{5}x}{8x^{1/5}(x-1)^{3/5}} = \frac{\frac{6}{5}x - \frac{4}{5}}{8x^{1/5}(x-1)^{3/5}}
\]

\[f'(x) = 0 \Rightarrow \text{num} = 0, \quad \frac{6}{5}x - \frac{4}{5} = 0 \quad \Rightarrow \quad x = \frac{2}{3}
\]

\[f'(x) \text{ DNE } \Rightarrow \text{den} = 0, \quad \frac{6}{5}x - \frac{4}{5} = 0 \quad \Rightarrow \quad x = 0, 1
\]

CP: \( x = 0, \frac{2}{3}, 1 \)

28. (5 pts) Find the absolute maximum and minimum of \( f(x) = x^3 - 6x^2 + 9x + 2 \) on \([-1, 4]\)

\[
f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3)
\]

\[
= 3(x - 3)(x - 1)
\]

CP: \( x = 1, 3 \), both are in interval.

candidates for location: \( x = -1, 1, 3, 4 \)

\[
f(-1) = (-1)^3 - 6(-1)^2 + 9(-1) + 2 = -1 - 6 - 9 + 2 = -14
\]

\[
f(1) = 1^3 - 6 + 9 + 2 = 6
\]

\[
f(3) = 3^3 - 6 \cdot 3^2 + 9 \cdot 3 + 2 = 27 - 54 + 27 + 2 = 2
\]

\[
f(4) = 4^3 - 6 \cdot 4^2 + 9 \cdot 4 + 2 = 64 - 96 + 36 + 2 = 6
\]

abs max \( y = 6 \) at \( x = 1 \) and \( x = 4 \)

abs min \( y = -14 \) at \( x = -1 \)
29. (5 pts) Use the Mean Value Theorem to find a value of \( f'(c) \) that is guaranteed to occur in \([1, 4]\) for \( f(x) = x^3 + 2x - 1 \).

\[
\begin{align*}
  f(1) &= 1^3 + 2 - 1 = 2, \\
  f(4) &= 4^3 + 8 - 1 = 71, \\
  f'(c) &= \frac{71 - 2}{4 - 1} = \frac{69}{3} = 23.
\end{align*}
\]

30. (5 pts) If \( 2 \leq f'(x) \leq 4 \) on \([0, 6]\) and \( f(0) = 4 \), how small or large can \( f(6) \) be?

\[
\begin{align*}
  2 &\leq \frac{f(6) - f(0)}{6 - 0} \leq 4 \\
  2 &\leq \frac{1}{6} (f(6) - 4) \leq 4 \\
  12 &\leq f(6) - 4 \leq 24 \\
  14 &\leq f(6) \leq 28.
\end{align*}
\]
31. (5 pts) Let \( y = 4x^3 + 3x^2 - 6x + 2 \). Find the intervals where \( y \) is increasing, decreasing, concave up, concave down. Identify the critical points, inflection points and relative extrema. Use this information to sketch a graph of the function.

\[
y' = 12x^2 + 6x - 6 = 6(2x-1)(x+1) \\
y'' = 24x + 6 = 6(4x+1)
\]

Critical Points (CP): \( x = -1, 1/2 \)
Inflection Points (IP): \( x = -1/4 \)

<table>
<thead>
<tr>
<th>Interval</th>
<th>( y' )</th>
<th>( y'' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-\infty, -1)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(-1, 1/2)</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>(1/2, 1)</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>(1, \infty)</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

Draw the curve first, and then mark the CP, IP.

32. (5 pts) Sketch \( f(x) \) if \( f'(3) = 0 \), \( f \) is increasing on \((\infty, 1) \cup (3, \infty)\) and decreasing on \((1, 3)\), and concave up on \((\infty, 1) \cup (1, \infty)\).

This implies \( f''(1) \) DNE; otherwise, we'd say \((-\infty, 1) \cup (1, \infty)\)

Again, sketch first then mark the points.