1. Draw a rough sketch of a function that has a removable discontinuity at \( x = 1 \), a jump discontinuity at \( x = 2 \) and is continuous from the left there, a jump discontinuity at \( x = 3 \) and is continuous from the right there, an infinite discontinuity at \( x = 4 \), a jump discontinuity at \( x = 5 \) without being continuous from the left or from the right there, and a kink at \( x = 6 \). There are many correct possible answers.

\[ \frac{(x-1)(x-3)}{(x+2)(x-5)} \rightarrow \frac{3-7}{3+2} = -\frac{4}{5} \]

2. Evaluate \( L = \lim_{x \to 3} \frac{x^2 - 10x + 21}{x^2 - x - 6} \). Show your algebraic work.

3. A machinist is required to make a circular disk of area \( 25\pi = 78.5398 \text{ cm}^2 \), which requires a radius of exactly 5 cm. (a) If the error tolerance for the area is ±2 cm\(^2\) (i.e., the area is 78.5398 ± 2), find the range of radii (smallest and largest radii) that a disk might have and still have an acceptable area. The area of a circle is \( A = \pi r^2 \). Use 4 digits after the decimal point in your calculations.

\[ b: \pi b^2 = 80.5398 \quad b = \sqrt{\frac{80.5398}{\pi}} = 5.0433 \]

\[ c: \pi c^2 = 76.5398 \quad c = \sqrt{\frac{76.5398}{\pi}} = 4.9359 \]

\[ \text{range:} \quad 4.9359 \leq r \leq 5.0433 \]

(b) In terms of the generic definition of a limit, \( \lim_{x \to a} f(x) = L \), identify numerical values for \( r, \delta, a \) and \( L \).

\[ \varepsilon = 0.002 \]

\[ \delta = 0.0430 \]

\[ a = 5 \]

\[ L = 25\pi \]
4. Differentiate \( f(x) = \frac{9}{48x^{13/19}} - \frac{1}{\sqrt[3]{x} + x^{2/7} + x^b} \)

\[
f' = \frac{2}{48} - \frac{12}{19} x^{31/19} + \frac{1}{2} x^{2/7} + 287x + 0
\]

5. Let \( f(x) = \begin{cases} 2x + 4, & \text{if } x < 2; \\ k, & \text{if } x = 2 \ (k \text{ will be determined in parts b and c}); \\ 3x - 8, & \text{if } x > 2; \end{cases} \)

a) State the formal definition of the continuity of a function \( f \) at the point \( x = a \).

\[
f \text{ is cont at } x = a \text{ if } \lim_{x \to a} f(x) = f(a)\]

b) What value of \( k \) should be used to make \( f \) continuous from the left at \( x = 2 \) (note that \( f(2) = k \))?

\[
2x + 4 \bigg|_{x=2} = 8
\]

5a: 3 pts

5b: 3 pts

5c: 3 pts

6. Use the Intermediate Value Theorem to show that \( f(x) = x^5 - 7 \) has at least one real root. Find an interval in which the root lies, and be sure to verify that each hypothesis of the Theorem is satisfied.

\[
2. \quad f \text{ is cont since it's a poly}
\]

\[
2. \quad f(0) = -7
\]

\[
2. \quad f(2) = 32 - 7 = 25
\]

\[
1. \quad \text{so } f \text{ has a root in } [0, 2]
\]

6: 7 pts
7. Differentiate \( g(r) = \frac{r^5 + 2r}{r^4 + 1} \). Simplify the numerator.

\[
g'(r) = \frac{(r^4 + 1) \left[ 5r^4 + 2 \right] - (r + 2r) \left[ 4r^5 \right]}{(r^4 + 1)^2}
\]

\[
= \frac{5r^8 + 5r^4 + 2r^4 + 2 - 4r^9 - 8r^4}{(r^4 + 1)^2}
\]

8. Differentiate \( f(x) = \sin(x^2 + \tan(x^3)) \). Do not simplify.

\[
f'(x) = \cos(x^2 + \tan(x^3)) \cdot \left[ 2x + \sec^2(x^3) - 3x^2 \right]
\]

9. Differentiate \( f(x) = \frac{x}{\sqrt{x^2 + 1}} \). Simplify completely – it works out to be surprisingly simple.

\[
f'(x) = \frac{(x + 1)^{1/2} \left[ 1 \right] - x \left[ \frac{x}{(x^2 + 1)^{3/2}} \right]}{(x + 1)^{3/2}}
\]

\[
= \frac{1}{(x^2 + 1)^{3/2}}
\]
10. a) State the formal definition of the derivative of \( f(x) \) at \( x = a \).

The derivative of \( f \) at \( x = a \) is
\[
\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}, \quad \text{provided the limit exists}
\]

b) Use the definition to find \( f'(x) \) for \( f(x) = \frac{1}{\sqrt{x}} \).

\[
f(x) = \frac{1}{\sqrt{x}}
\]
\[
f(x+h) = \frac{1}{\sqrt{x+h}}
\]
\[
f(x+h) - f(x) = \frac{1}{\sqrt{x+h}} - \sqrt{x}
\]
\[
= \frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x+h} \sqrt{x}}
\]
\[
= \frac{x - (x+h)}{\sqrt{x+h} \sqrt{x}}
\]
\[
= -\frac{1}{\sqrt{x+h} \sqrt{x}}
\]
\[
f'(x) = \lim_{h \to 0} \left( -\frac{1}{\sqrt{x+h} \sqrt{x}} \right) = -\frac{1}{(x)(2\sqrt{x})} = -\frac{1}{2x^{3/2}}
\]

11. Find the derivative of \( h(t) = (t^2 + t)(t^3 + 2t)(t^4 + 2t) \). Do not simplify.

\[
h'(t) = \left[ 2t + 1 \right] \left( t^3 + t^2 \right) \left( t^4 + 2t \right) + \left( t^2 + t \right) \left[ 3t^2 + 2t \right] \left( t^4 + 2t \right) + \left( t^2 + t \right) \left( t^3 + t^2 \right) \left[ 4t^3 + 2 \right]
\]

12. Evaluate \( \lim_{x \to -1} \frac{x}{x+1} \). It is especially important to show your work on this problem.

\[
\begin{align*}
\text{VA} & \quad \text{let } x = -1 + h \\
\frac{-1 + h}{(-1 + h) + 1} & \quad \text{let } x = -1 + h \\
\frac{-1 + h}{h} & \quad \text{let } x = -1 + h \\
L & = \text{do}
\end{align*}
\]