3450:730 Advanced Numerical Solution of PDEs, Kreider, Summer 2008

Computing Project 2 (50 points)

Due date: Thursday 19 June

Consider this Burger's Equation problem:

$$\begin{aligned} u_t &= \alpha u_{xx} - u u_x \qquad 0 < x < 1, t > 0 \\ u(x,0) &= \sin(\pi x) \\ u(0,t) &= 0 \\ u(1,t) &= 0 \end{aligned}$$

Use the results of Homework 10 to implement the Crank-Nicolson method with successive approximations to solve this problem. You will write 2 versions of the code. In the first version, keep the u_i^{n+1} from u_t on the right side, written as $v_i^{(m)}$ (Part A). In the second version, bring this term to the left and incorporate it into the matrix (Part B). In each version, use the following parameters:

$\alpha = .01$, $N = imax = 1001$, $dt = 1.0d-5$, $mmax = 50$, $tol = 1.0d-12$

For the first version, which is unstable, use $kmax = 2$ and print the value of err for each iteration so that you can see the instability. For the second version, use $kmax = 10^4$ and suppress the err print so you don't generate too much needless output. The successive approximation loop converges in 1, 2 or 3 iterations at each time step with this value of tol. To make the grading easier, please use the ℓ_2 norm rather than the ℓ_1 norm to compute err.

This took about 215 seconds on my old machine (a 600 MHz Pentium) and about 4 seconds on my new machine (a 3.2 GHz Pentium), so you should see run times in that range.

Turn in the following:

Version 1: copy of the code and output showing values of err.

Version 2: copy of the code, the first 10 and last 10 output values of u_i^{kmax} (the whole list takes too many pages, but if I have a few of the values, I can grade more easily) and, most importantly, a plot of u_i^{kmax}.

Correct code is 40 points, style and presentation of code and results is 10 points.