

Computing Project 1 (50 points)

Due date: Thursday 12 June

Consider the diffusion equation

$$u_t = Du_{xx} + bu_x + S(x, t) \quad 1 < x < 3, t > 0 \quad (1)$$

$$u(x, 0) = \exp(-50(x - 2)^2) \quad (2)$$

$$u(1, t) = 1 \quad (3)$$

$$u_x(3, t) = -25 \quad (4)$$

$$D = 3 \quad (5)$$

$$S(x, t) = 800 \sin(5\pi x) \exp(-t) \quad (6)$$

$$b = 10^{-4} \quad (7)$$

Write a Crank-Nicolson code to solve the problem using $N = 1001$ spatial points (x_1, \dots, x_{1001}) . Run the code 4 times:

1. with $dt = 2dx$ and $kmax = 50$ time steps (so $t_0 = 0$ and $t_{50} = 1$)
2. with $dt = 4dx/3$ and $kmax = 75$ time steps (so $t_{75} = 1$)
3. with $dt = dx$ and $kmax = 100$ time steps (so $t_{100} = 1$)
4. with $dt = dx/5$ and $kmax = 500$ time steps (so $t_{500} = 1$)

Note that you can build your matrix (vectors AL, AM, AR) outside the time loop, but the righthand side must be rebuilt at each time step, so it goes inside the loop. Use your tridiagonal solver to solve the system.

The only output for each run should be the values of u_i^{kmax} . Output these values to different data files, and plot the graphs together. The results should be very similar, but you should notice some slight differences. Print 2 plots, one showing the global solution on $(1, 3)$ and one zoomed in to show the differences caused by refining the grid. If you wish, you can practice by also producing a surface plot, but it's not necessary in this first assignment. Turn in a copy of your code, the 2d graphs and a representative copy of any input files that you may have used.

There are various ways to implement the 4 different cases, in terms of specifying the grid spacings and output file names. After the assignment, I'll show you my version, and we'll see if you were able to find a better way to handle it.

I will grade the projects for accuracy (40 points) and style (10 points). The code should be well organized and easily readable. Documentation should be sparse – short comments to remind you what you are doing in each section of the code. I will be picky about the style you use, simply because I am trying to teach you a set of conventions that will improve your code-writing abilities.

Notes: Crank-Nicolson does not work if b is too large. Highly convective flows are difficult to deal with. We will discuss this issue a bit when we cover hyperbolic PDEs. Also note that with b small, you can make the time steps relatively big here and still get good solutions.