3450:730 Advanced Numerical Solution of Partial Differential Equations, Kreider, Summer 2008

Homework Problem 9

Due date: Thursday 5 June 2008

9. (20 points) Consider the diffusion equation

$$u_t = Du_{xx} + bu_x + S(x,t) \tag{1}$$

on the domain $1 < x < 3$, $t > 0$ with boundary conditions $u(1,t) = A$ and $u_x(3,t) = B$, and initial condition $u(x,0) = \exp(-50 * (x-2)^2)$.

Discretize the equation using the Crank-Nicolson scheme to obtain the standard interior equation used to build the matrix. Use second order central differences for the convection term (bu_x). Treat b as a constant.

Discretize the left boundary condition to obtain the first equation in the system. Discretize the right boundary condition and use a ghost point to obtain the final equation in the system.

Write a schematic of the matrix, including the righthand side, assuming that the spatial grid is given by $(x_1, \ldots, x_N)$ with spacing dx. Pay particular attention to the first and last rows, which correspond to the boundary conditions. Use symbols α^-, α^+, etc to make it easier to write. This schematic will be the main resource for Computing Project 1.

In the first programming assignment, you will write a code to solve a problem of this form. The point here is to write your equations on paper using notation that generalizes in an easy fashion. You will be using your tridiagonal solver.

Another thing I should mention is that the tridiagonal algorithm is sometimes called the Crout algorithm by mathematicians, and is called the Thomas algorithm by engineers.