

Homework Problem 11

Due date: Tuesday 17 June 2008

9. (20 points) Consider the problem

$$u_t = \alpha(u_{xx} + u_{yy}) + Au_x + Bu_y + Cu + S(x, y, t) \quad 0 < x < 1, 0 < y < 2, t > 0 \quad (1)$$

$$u(x, 0, t) = Q(x, t) \quad (2)$$

$$u(x, 2, t) = G(x, t) \quad (3)$$

$$u(0, y, t) = -\beta u_x(0, y, t) \quad (4)$$

$$u(1, y, t) = H(y, t) \quad (5)$$

$$u(x, y, 0) = F(x, y) \quad (6)$$

In Computing Project 3, you will solve this using the Peaceman-Rachford ADI scheme. In that project, specific values of the parameters and boundary conditions will be given.

This assignment is to write the two iteration equations for the PR method in the form indicated below. To standardize the notation, please use $u(i, j)$ for u_{ij}^n , $v(i, j)$ for $u_{ij}^{n+1/2}$ and $w(i, j)$ for u_{ij}^{n+1} . Also use the symbol rx for $\frac{\alpha dt}{dx^2}$ and ry for $\frac{\alpha dt}{dy^2}$.

For the first equation, set $j = 1$ and then write the equations for $i = 1, i = 2, \dots, imax - 1$ and $i = imax$ separately. Then set $j = 2, \dots, jmax - 1$ and write the equations for $i = 1, i = 2, \dots, imax - 1$ and $i = imax$ separately. Finally, set $j = jmax$ and write the equations for $i = 1, i = 2, \dots, imax - 1$ and $i = imax$ separately. This way, you'll clearly see where and how you handle the corners, where you use ghost points, where the tridiagonal matrix enters the drama, and where you simply assign the Dirichlet boundary conditions without resorting to a matrix formulation.

For the second equation, do the same thing as above, but switch the roles of i and j .

All of our lives will be much simpler if you follow my suggested format. For the first equation, first write the generic discretization, and use the symbols AAL, AAM, AAR for the matrix coefficients. Then write

```

Let j=1.
  for i=1, use v_{ij} = Q_i^{n+1/2}
  for i=2,imax-1, use v_{ij} = Q_i^{n+1/2}
  for i=imax, use v_{ij} = ( Q_i^{n+1/2} + H_j^{n+1/2} ) / 2
Let j=2:jmax-1
  for i=1, use ...
  for i=2,imax-1, use ...
  for i=imax, use ...
Let j=jmax
  for i=1, use ...
  for i=2,imax-1, use ...
  for i=imax, use ...

```

For the second equation, use the symbols BAL, BAM, BAR for the matrix coefficients, and follow a similar format.