Chapter 8: Voting and Social Choice

Section 8.1: Measuring voting power: Does my vote count?

Voting Coalition: A group of voters who vote the same way.

Winning Coalition: A set of voters with enough votes to determine the outcome of an election, otherwise it is a Losing Coalition.

Quota: The number of votes necessary to win the election, in a voting system.

Example: Suppose there are three delegates to a county convention: Abe has 4 votes from his precinct, Ben has 3 votes, and Condi has 1 vote. A simple majority of the votes wins.

1. What is the quota?

2. Make a table listing all of the coalitions of voters. Designate which of them are winning coalitions.

Critical voter: A member of a winning coalition is a critical voter if the coalition becomes a losing coalition when that voter is removed.

Example: The table below contains information from the three winning coalitions in the previous example. Who is a critical voter in each coalition?

<table>
<thead>
<tr>
<th>Number of Votes</th>
<th>Total Votes</th>
<th>Winning Coalition?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abe 4 Ben 3 Condi 1</td>
<td>8</td>
<td>Yes</td>
</tr>
<tr>
<td>Abe 4 Ben</td>
<td>7</td>
<td>Yes</td>
</tr>
<tr>
<td>Abe Condi</td>
<td>5</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Summary: Winning Coalitions and Critical Voters: A set of voters with enough votes to determine the outcome of an election is a winning coalition. A voter in a winning coalition is critical for that coalition if the coalition is no longer a winning one when that voter is removed. We can summarize the essential information about coalitions in a coalition table.

Example: If there are 7 voters, there are $2^7$ possibilities for voters to be in or not in a coalition. This includes the possibility of all the voters not being in any coalition, so there are $2^7 - 1$ possibilities.

The Banzhaf power index: The number of times a voter is critical in a winning coalition divided by the total number of instances in which any voter is critical. Expressed as a fraction or percentage.

Example: Compute the Banzhaf index for each county convention delegate described in the previous examples.
Example: At the 1988 Democratic National Convention, Michael Dukakis’s delegates had 1401 votes, Jesse Jackson’s had 1218 votes, Al Gore’s had 325 votes, and Bruce Babbitt’s had 197 votes. It is common that candidates form coalitions. A simple majority is required to win the nomination.

1. What is the quota?
2. Determine the winning coalitions.
3. Determine the critical voters (candidates) in each winning coalition.
4. Determine the Banzhaf index of each candidate.

Swing voter: Supposing the voters vote in order and their votes are added as they vote, the swing voter is the voter whose votes make the total meet the quota and thus decide the outcome. Which is the swing voter depends on the order the votes are cast.

Example: Members of the European Union have votes on the Council determined roughly by a country’s population but progressively weighted in favor of smaller countries. Ireland has 7 votes, Cyprus has 4 votes, and Malta has 3 votes. Supposing a simple majority wins, make a table with all the permutations of voters and the swing voter in each case.

The Shapley-Shubik power index is calculated as the fraction (or percentage) of all permutations of the voters in which that voter is the swing.

Example: Compute the Shapley-Shubik power index for the committee of Ireland, Cyprus, and Malta from the previous example.

Example: In the 2004 election, the votes from Florida, Michigan, Ohio, and Pennsylvania were important in determining the outcome. Florida had 27 electoral votes, Michigan had 17, Ohio had 20, and Pennsylvania had 21. Assume a majority from only these four states would determine the election. In this case the quota is 43.

1. How many permutations of these four states are there?
2. Make a table listing each case and its swing voter.
3. Find the Shapley-Shubik index for each state.

Example: Ace Solar (page 540): Suppose there are four stockholders in a meeting of the Ace solar Corporation and that each person gets as many votes as the number of his or her shares. Assume that Abe has 49 shares, Ben has 48 shares, Condi has 4 shares, and Doris has 3 shares.

#24: Assume that a simple majority is required to prevail in a vote. Make a table listing all the permutations of the voters and the swing voter in each case, and calculate the Shapley-Shubik index for each voter.

Make a table listing all the winning coalitions and critical voter in each case, and calculate the Banzhaf index for each voter.

#25: Assume now that a two-thirds majority is required to prevail in a vote, so the quota is 70. Calculate the Shapley-Shubik index for each voter.

Calculate the Banzhaf index for each voter.

Practice Problems: 1, 3, 5, 7, 9, 17–23, 27–31, 35–36