

File 805

Diff Eq - 3450:235  
EXAM #1 Spring 99  
Show ALL your work.

NAME \_\_\_\_\_  
ROW \_\_\_\_\_

100 Points

1. Solve  $x \frac{dy}{dx} + 4y = \frac{1}{x^4(x-1)}$ .

15 Points

2a. A tank holds 400 gallons of water. Initially the tank is filled with 300 gallons of water with 50 kg of chlorine in solution. Water containing 2 kg of chlorine per gallon is entering the tank at the rate of 3 gal/min. The well-stirred mixture is flowing out of the tank at the rate of 1 gal/min. If  $C(t)$  denotes the amount of chlorine in kg in the tank at any time  $t$  in minutes, find a differential equation and initial condition which governs the amount of chlorine in the tank at any time  $t$ . DO NOT SOLVE THE EQUATION.

10 Points

2b. Suppose that you know the solution  $C(t)$  to the above differential equation. How would you use this solution to find the amount of chlorine in the tank when the tank begins to overflow?

5 Points
30 Points

Solve  $ydx - (x + \sqrt{xy})dy = 0$  subject to  $y(0) = 1$ .

15 Points

Solve  $x \frac{dy}{dx} = 6x^2\sqrt{y} - 2y$ .

15 Points

30 Points

5. Use the substitution  $y(x) = x + \frac{1}{v(x)}$  to solve the equation  $\frac{dy}{dx} = 2x^2 + \frac{y}{x} - 2y^2$ .

This substitution should result in a simpler equation for  $v(x)$ . Now solve for  $v(x)$ .

6. A man has a fortune which he is spending at a rate that is proportional to the square of his present wealth. If he had \$1 million a year ago and has \$1/2 million today, how much will he be worth in 9 years?

15 Points

7a. Suppose the temperature of a cup of coffee obeys Newton's Law of Cooling. The cup of coffee initially at  $95^{\circ}\text{C}$  and cools to  $80^{\circ}\text{C}$  in 5 minutes while sitting in a room of temperature  $21^{\circ}\text{C}$ . If  $T(t)$  denotes the temperature of the coffee at any time  $t$  in minutes, find a differential equation and condition which govern the temperature at any time  $t$ . DO NOT SOLVE THE DIFFERENTIAL EQUATION.

5 Points

7b. Suppose you know the solution  $T(t) = 74 e^{(1/5 \ln(59/74))t} + 21$  for the temperature of the coffee. When will the coffee be  $50^{\circ}\text{C}$ ? What is the steady state temperature of the coffee?

5 Points

25 Points