1. Solve \( (x^2 + 1)\frac{dy}{dx} = 6x - 3xy \) subject to \( y(0) = 7 \).

2a. When light passes through the water of a lake, the rate at which the intensity \( I \) of the light decreases with depth \( x \) into the lake is proportional to the intensity \( I(x) \) of the light at any depth \( x \). At the surface of the lake, the intensity of the light is measured to be the constant value \( I_0 \). Three feet below the surface the intensity is 75\% of \( I_0 \). SET UP BUT DO NOT SOLVE the differential equation(s) and condition(s) necessary to find the intensity of the light at any depth.

2b. Without solving the above differential equation(s) and using the variable names listed above, state how would you determine the depth at which the intensity is 25\% of that at the surface.
3. Solve \( \frac{dy}{dx} = 3xy^{4/3} - 6y \).

4. Solve \( \frac{dy}{dx} = y + \sqrt{x^2 - y^2} \).
5. Solve \((6xy - y^3)dx - (3xy^2 - 3x^2 - 4y)dy\).

6. Solve \(\frac{x}{y} \frac{dy}{dx} - 4x^2 + 2 \ln y = 0\) by making the substitution \(z = \ln y\).
7a. A motorboat has a mass of \( \frac{1000 \text{ lb}}{\text{ft/sec}^2} \). The boat starts from rest at the dock and travels in a straight line. The boat's motor provides a thrust of 5000 lb. The water leads to a drag resistance that is proportional to the velocity, \( V(t) \), of the boat. The proportionality constant of the drag is \( 100 \frac{\text{lb}}{\text{ft/sec}} \). SET UP BUT DO NOT SOLVE differential equation(s) and condition(s) to determine the distance, \( D(t) \), of the boat from the dock at any time \( t \).

9 Points

7b. Without solving the above differential equation(s) and using the variables described above, state how you would determine the velocity of the boat when it is 10 miles from the dock.

5 Points

14 Points