1. Find the general solution to \( \frac{d^2y}{dx^2} - 4 \frac{dy}{dx} = 4x + e^x \).

2. A 96 lb mass is attached to a spring hanging from the ceiling. This causes the spring to stretch 8 ft on coming to rest at equilibrium. The damping constant for the system is 18 \( \frac{lb \cdot \text{sec}}{ft} \).

At time \( t = 0 \) sec the mass is pulled down 6 inches below the equilibrium point and given an upward velocity of 4 ft/sec. The motion of the mass is further driven by an external force of \( 12 \cos(2t) + 3 \sin(2t) \) lbs. Write down the governing differential equation and initial conditions for the motion of the mass. DO NOT SOLVE THE EQUATION.
3. Find the general solution to \(3y'' + 27y = 18 \csc(3x)\).

4. One solution of the equation \(4(x - 3)^2y'' + 8(x - 3)y' + y = 0\) is \(y = (x - 3)^{-1/2}\). Find the general solution to this same equation.
5. A solution to \( x^2y'' - 3xy' + 3y = 2x^4e^x \) is \( y = 2x^2e^x - 2xe^x \). Solve the following problem:
\( x^2y'' - 3xy' + 3y = -6x^4e^x \), \( y(1) = 0 \), \( y'(1) = 8e \).

6. Solve the system of equations:
\[
\begin{align*}
\frac{dx}{dt} & = \frac{1}{2}x + 9y \\
\frac{dy}{dt} & = \frac{1}{2}x + 2y
\end{align*}
\]
A mass of $M$ slugs ($\frac{\text{lb} - \text{sec}^2}{\text{ft}}$) is attached to a spring with a spring constant of $6 \frac{\text{lb}}{\text{ft}}$ and a damping constant of $\frac{1}{2} \frac{\text{lb} - \text{sec}}{\text{ft}}$. An external force of $10 \cos(3t)$ lbs is applied to this system. The initial conditions for the motion $x(t)$ of the mass are $x(0) = 2$, and $x'(0) = 0$. The equation of motion for the mass is found to be

$$x(t) = e^{-\sqrt{2}t} \left[ -\frac{4}{3} \cos \left( \frac{\sqrt{47}}{2} t \right) - \frac{64}{3\sqrt{47}} \sin \left( \frac{\sqrt{47}}{2} t \right) \right] + \frac{10}{3} \left[ \cos(3t) + \sin(3t) \right].$$

a) Find the numerical value of the mass $M$.

b) What value of the damping constant leads to critical damping?

c) Find the amplitude and phase shift of the steady-state motion of the mass.