.70 ~~3/14/03~~ 10/28/03 File # 805

Diff Eq - 3450:235
EXAM # 2 Spr 98

NAME______________________
ROW______________________

100 Points

Show **ALL** your work.

1. Find the general solution to $\frac{d^4y}{dx^4} - \frac{d^2y}{dx^2} = 4x + e^x$.

15 Points

2. A 96 lb mass is attached to a spring hanging from the ceiling. This causes the spring to stretch 8 ft on coming to rest at equilibrium. The damping constant for the system is 18 $\frac{\text{lb} - \text{sec}}{\text{ft}}$.

At time t = 0 sec the mass is pulled down 6 inches below the equilibrium point and given an upward velocity of 4 ft/sec. The motion of the mass is further driven by an external force of 12 cos(2t) + 3 sin(2t) lbs. Write down the governing differential equation and initial conditions for the motion of the mass. DO NOT SOLVE THE EQUATION.

10 Points

25 Points

3. Find the general solution to $3y'' + 27y = 18 \csc(3x)$.

15 Points

4. One solution of the equation $4(x-3)^2y'' + 8(x-3)y' + y = 0$ is $y = (x-3)^{-1/2}$.
Find the general solution to this same equation.

15 Points

30 Points

5. A solution to $x^2y'' - 3xy' + 3y = 2x^4e^x$ is $y = 2x^2e^x - 2xe^x$. Solve the following problem:
$x^2y'' - 3xy' + 3y = -6x^4e^x$, $y(1) = 0$, $y'(1) = 8e$.

15 Points

6. Solve the system of equations: $\frac{dx}{dt} = \frac{1}{2}x + 9y$
$\frac{dy}{dt} = \frac{1}{2}x + 2y$

15 Points

30 Points

7. A mass of M slugs $\left(\frac{\text{lb}-\text{sec}^2}{\text{ft}}\right)$ is attached to a spring with a spring constant of 6 $\frac{\text{lb}}{\text{ft}}$ and a damping constant of $\frac{1}{2}\ \frac{\text{lb}-\text{sec}}{\text{ft}}$. An external force of 10 cos(3t) lbs is applied to this system. The initial conditions for the motion x(t) of the mass are x(0) = 2, and x′(0) = 0. The equation of motion for the mass is found to be

$$x(t) = e^{-t/2}\left[\frac{-4}{3}\cos\left(\frac{\sqrt{47}}{2}t\right) - \frac{64}{3\sqrt{47}}\sin\left(\frac{\sqrt{47}}{2}t\right)\right] + \frac{10}{3}[\cos(3t) + \sin(3t)].$$

a) Find the numerical value of the mass M.

5 Points

b) What value of the damping constant leads to critical damping?

2 Points

c) Find the amplitude and phase shift of the steady-state motion of the mass.

8 Points

15 Points