

Diff Eq - 3450:335
Final S04

NAME _____
ROW _____

1
150 Points

Show ALL your work.

1. Find the general solution to $\frac{d^2y}{dt^2} + 4y = \cos(2t)$.

10 Points

2. Use Laplace transforms to find $x(t)$ ONLY, where $x(0) = 0$, $x'(0) = 0$, $y(0) = 0$, $y'(0) = 0$, and

$$x' - x + y' = t$$
$$x' + x - y'' = 0.$$

15 Points

25 Points

3. Use the Laplace transform technique to solve $y' + y = f(t)$ given $y(0) = 3$

$$\text{and } f(t) = \begin{cases} 0, & 0 \leq t < 1 \\ 5e^{-t}, & t \geq 1 \end{cases}.$$

15 Points

4. One solution to $xy'' - 2y' + 9x^5y = 0$ is $y = \sin(x^3)$. Find the other fundamental solution.

10 Points

5. The general solution to $x^2y'' + 2xy' - 2y = 0$ is $y = c_1x + c_2x^{-2}$. Find $f(x)$ if a particular

solution to $x^2y'' + 2xy' - 2y = f(x)$ is $y_p = \frac{x}{3} \int \frac{\cos(x)}{x^3} dx - \frac{1}{3x^2} \int \cos(x) dx$.

10 Points

35 Points

6. Use Laplace transforms to solve the differential equation $y'' + 4y' + 4y = 56t^6e^{-2t}$ subject to $y(0) = 1, y'(0) = 0$.

15 Points

7. Find the general solution to $x \frac{dy}{dx} + (1+x)y = 4x^3e^{-x}$.

10 Points

8. The eigenvalues of $\begin{bmatrix} 3 & -18 \\ 2 & -9 \end{bmatrix}$ are $\lambda = -3, -3$.

Solve the system of differential equations $\frac{dx}{dt} = 3x - 18y$

$$\frac{dy}{dt} = 2x - 9y.$$

10 Points

35 Points

9. Given the differential equation $y'' + 3y' + 2y = f(t)$ subject to $y(0) = 0, y'(0) = 3$, the Laplace transform $Y(s) = \mathcal{L}[y] = \frac{3s}{(s+1)(s+2)^2}$. Find $f(t)$.

10 Points

10. A solution to $y'' + 4y' + 3y = 3x^2 + 8x + 2 + 2e^{-x}$ is $y = x^2 + xe^{-x}$. Find the solution to $y'' + 4y' + 3y = 3x^2 + 8x + 2 + 2e^{-x}$ subject to $y(0) = 0, y'(0) = 3$.

10 Points

11. Solve $\frac{1}{x^2} \frac{dy}{dx} - 3 \cos(x^3) y^3 = 0$ subject to $y(0) = 1$.

10 Points

30 Points

12. Find a series solution about $x = 0$ for the differential equation $y'' - xy' - 2y = 0$.

15 Points

13. Use the transformation $z = y^{1/2}$ to transform the Bernoulli equation $y' - xy = x^2\sqrt{y}$ into a linear equation in *standard form*.

10 Points

25 Points