

METHODS OF APPLIED MATHEMATICS I and II
3450:633, 634
Tentative Outline

- I. Introduction
 - A. What is Applied Mathematics?
 - B. Techniques of Applied Mathematics
 - 1. Transformations
 - 2. Approximation

- II. Review of Vector Spaces
 - A. Linear Combinations and Bases
 - B. Inner Products and Norms
 - C. Metrics
 - D. Hilbert Spaces - L^2 and l^2
 - E. Approximation in Hilbert Spaces
 - F. Orthogonal Functions and Fourier Series
 - G. Separation of Variables Solutions to PDEs
 - H. Sturm-Liouville Eigenvalue Problems

- III. Spectral Theory for Matrices
 - A. Adjoints
 - B. Self-Adjoint Matrices
 - C. Spectral Decomposition
 - D. Fredholm Alternative Thm

- IV. Ordinary Differential Equations
 - A. Differential Operators
 - 1. Adjoint Problem
 - 2. Fredholm Alternative Theorem
 - B. Eigenfunctions and Expansions
 - 1. Homogeneous Boundary Condition Problems
 - 2. Inhomogeneous Boundary Condition Problems
 - 3. Sturm-Liouville Problems
 - a. Orthogonal Polynomials
 - b. Bessel Functions

- V. Introduction to Asymptotic and Perturbation Methods
 - A. Simple Examples
 - B. Physical Examples

- VI. Basic Asymptotic Concepts
 - A. Definitions
 - B. Error Function Example

- VII. Review of Partial Differential Equations
 - A. Gauss Divergence Theorem
 - 1. Green's first and second identities
 - B. Heat equation
 - C. Laplace equation
 - D. Wave equation
 - E. Other equations

- VIII. Differential Equations
 - A. Straight Forward Expansion
 - B. Poincare-Linstedt Method
 - 1. Phase Plane
 - 2. Van der Pol and Rayleigh Oscillator
 - C. Multiple Scales
 - 1. Scaling the Dependent Variable

- D. Singular Perturbations and Matched Asymptotic Expansions
 - 1. Linear Equations
 - 2. General Theory - 2nd Order Equations
 - 3. Multiple Boundary Layers
 - 4. Important Differential Equations of Mathematical Physics
 - 5. Nested Boundary Layers
 - 6. Internal Boundary Layers
 - 7. Nonlinear Examples
- E. WKB Theory

- IX. Asymptotic Expansion of Integrals
 - A. Laplace's Method and Watson's Lemma
 - B. Method of Stationary Phase
 - C. Method of Steepest Descents

- X. Bifurcation and Linear Stability Analysis
 - A. Nonlinear Diffusion Equation Example
 - 1. Bifurcating Solutions
 - 2. Linear Stability
 - B. Transient Analysis - Weakly Nonlinear

- XI. Case Studies

- XII. Ordinary Differential Equations
 - A. Dirac Delta Functions and The Theory of Distributions
 - B. Green's Functions
 - C. Differential Operators
 - 1. Green's Functions and the Adjoint Problem
 - D. Integral Equations

- XIII. Green's Function Solutions
 - A. Laplace Equation
 - 1. Uniqueness
 - 2. General Formulation and Green's Second Identity
 - 3. Fundamental Singularities in 2D and 3D - Free Space Green's Functions
 - 4. Interior Problems
 - 5. Exterior Problems
 - 6. Method of Images
 - a. Circular Hole
 - b. Half and Quarter Planes
 - 7. Modal Decomposition
 - 8. Transform Techniques
 - 9. Mixed or Robin Problems
 - B. Wave Equation - Helmholtz Equation
 - 1. Incident, Reflected, and Scattered Fields
 - 2. Hankel Functions and the Free Space Green's Function
 - 3. Radiation Conditions
 - 4. Aperture Problem
 - 5. Wave Guide Problems
 - 6. Approximations - Born, small body, etc.
 - C. Heat Equation
 - 1. Causal Green's Function
 - 2. Adjoint Causal Green's Function - Backward Heat Equation
 - 3. Half and Quarter Plate Problems
 - 4. Transform Methods
 - 5. Stefan Problem