METHODS OF APPLIED MATHEMATICS I and II
3450:633, 634
Tentative Outline

I. Introduction
   A. What is Applied Mathematics?
   B. Techniques of Applied Mathematics
      1. Transformations
      2. Approximation

II. Review of Vector Spaces
   A. Linear Combinations and Bases
   B. Inner Products and Norms
   C. Metrics
   D. Hilbert Spaces - L^2 and l^2
   E. Approximation in Hilbert Spaces
   F. Orthogonal Functions and Fourier Series
   G. Separation of Variables Solutions to PDEs
   H. Sturm-Liouville Eigenvalue Problems

III. Spectral Theory for Matrices
   A. Adjoints
   B. Self-Adjoint Matrices
   C. Spectral Decomposition
   D. Fredholm Alternative Thm

IV. Ordinary Differential Equations
   A. Differential Operators
      1. Adjoint Problem
      2. Fredholm Alternative Theorem
   B. Eigenfunctions and Expansions
      1. Homogeneous Boundary Condition Problems
      2. Inhomogeneous Boundary Condition Problems
      3. Sturm-Liouville Problems
         a. Orthogonal Polynomials
         b. Bessel Functions

V. Introduction to Asymptotic and Perturbation Methods
   A. Simple Examples
   B. Physical Examples

VI. Basic Asymptotic Concepts
   A. Definitions
   B. Error Function Example

VII. Review of Partial Differential Equations
   A. Gauss Divergence Theorem
      1. Green's first and second identities
   B. Heat equation
   C. Laplace equation
   D. Wave equation
   E. Other equations

VIII. Differential Equations
   A. Straight Forward Expansion
   B. Poincare-Linstedt Method
      1. Phase Plane
      2. Van der Pol and Rayleigh Oscillator
   C. Multiple Scales
      1. Scaling the Dependent Variable

- D. Singular Perturbations and Matched Asymptotic Expansions
  1. Linear Equations
  2. General Theory - 2nd Order Equations
  3. Multiple Boundary Layers
  4. Important Differential Equations of Mathematical Physics
  5. Nested Boundary Layers
  6. Internal Boundary Layers
  7. Nonlinear Examples
- E. WKB Theory

IX. Asymptotic Expansion of Integrals
- A. Laplace's Method and Watson's Lemma
- B. Method of Stationary Phase
- C. Method of Steepest Descents

X. Bifurcation and Linear Stability Analysis
- A. Nonlinear Diffusion Equation Example
  1. Bifurcating Solutions
  2. Linear Stability
- B. Transient Analysis - Weakly Nonlinear

XI. Case Studies

XII. Ordinary Differential Equations
- A. Dirac Delta Functions and The Theory of Distributions
- B. Green's Functions
- C. Differential Operators
  1. Green's Functions and the Adjoint Problem
- D. Integral Equations

XIII. Green's Function Solutions
- A. Laplace Equation
  1. Uniqueness
  2. General Formulation and Green's Second Identity
  3. Fundamental Singularities in 2D and 3D - Free Space Green's Functions
  4. Interior Problems
  5. Exterior Problems
  6. Method of Images
     a. Circular Hole
     b. Half and Quarter Planes
  7. Modal Decomposition
  8. Transform Techniques
  9. Mixed or Robin Problems
- B. Wave Equation - Helmholtz Equation
  1. Incident, Reflected, and Scattered Fields
  2. Hankel Functions and the Free Space Green's Function
  3. Radiation Conditions
  4. Aperture Problem
  5. Wave Guide Problems
  6. Approximations - Born, small body, etc.
- C. Heat Equation
  1. Causal Green's Function
  2. Adjoint Causal Green's Function - Backward Heat Equation
  3. Half and Quarter Plate Problems
  4. Transform Methods
  5. Stefan Problem