1. **How big is the world? The ancient Greeks knew; you can, too!** Find out how Eratosthenes computed the size of the world. Feel free to discuss the history of some of the techniques required, such as surveying and trigonometry.

References: books on the history of mathematics.

2. **Gravitational pull as an explanation for astrology.** According to the theory (or "law") of gravity, all masses attract each other, and the strength of the attraction is given by a formula that you can look up. For example, you and the Earth attract each other, and that's why you can't jump very far.

Proponents of astrology sometimes suggest that the position of the planets at the time of your birth can have a subtle effect on your life because of the slight gravitational pull they exert on you. Using standard references works, find the masses of some of the nearby planets, find their typical distances from the Earth, and find the formula that allows you to compute the gravitational pull of these planets on a typical newborn baby. Compare these figures with the gravitational pull exerted on the baby by other objects, such as an obstetrician or a nervous father.

3. **Don't send get-well cards to Craig Shergold.** In 1989, a nine-year-old English boy with cancer wanted to get into the *Guinness Book of World Records* by receiving the most get-well cards. Soon, someone issued an appeal on the Internet, urging us all to send him cards and to pass on the message. Bad idea.

Craig was successfully treated in 1991, and last I heard was still healthy. However, the cards continue to pour in. Even early on, the cards were so numerous that Craig's house was assigned its own postcode. After they were too swamped by mail, Craig's family had to move. Some people sent cards to the Make-a-Wish Foundation instead, even though it has no connection with the Shergolds. It too is now also swamped, and has been begging people to stop for years, but to little avail.

Suppose Craig receives ten million cards in 2001. Suppose that each year thereafter, Craig receives 10% fewer cards than he did the previous year.

(a) How many cards will Craig receive during the ten-year period 2001–2010?
(b) How many cards will he receive during the twenty-year period 2001–2020?
(c) How many cards will Craig receive during the thirty-year period 2001–2030?
(d) How many cards will Craig receive during the forty-year period 2001–2040?

Based on your answers, find and justify an estimate for how many cards Craig will receive in all. Hint: In 70 years, Craig will still be receiving more cards per year than most people receive in a lifetime.

Moral: Never participate in chain letters.

Report on other examples of rumors, online petitions, etc., that will not die.

Sources: For information about urban legends, see `www.snopes.com`. For a quick way to compute the kinds of sums asked for (that is, the sum of a "geometric series"), see Chapter 10 of our textbook. For a more tedious but straightforward method, use a spreadsheet.

4. **Let's endow some scholarships** The home territory of the University of Akron consists of five counties. Wouldn't it be nice if someone were to set up a scholarship fund to pay the tuition of, say, the top 10% of each high school's graduating class? How much would this cost per year? You'll need to know how many people graduate in these counties, and find (and justify!) an estimate for how many of the top 10% would choose to attend the University of Akron if it were free.

Now suppose you wanted to establish an endowment to pay for these scholarships. The investment income from this endowment would have to grow over time, to take rising tuition rates into account. How much money would you have to deposit into this fund?

5. **How much is that car?** This one might require a calculator or spreadsheet, but it also might be particularly relevant to your life. Let's suppose that you're on the market for a used car, and you feel that you can afford to pay \$200 per month. A dealer offers you a \$10000 car with no money down, financing at the low rate of $\frac{1}{2}\%$ per month (which corresponds to just over 6% per year), and a \$200 monthly payment.

(a) Three years from now, you expect that the car will have lost half of its value, and will thus be worth \$5000. How much will you still owe on your car?
(b) When will your car be all paid for?
(c) Suppose the dealer says, "Just because I like you, I'll set up a deal where you only have to pay \$75 per month." How does this change your answers to the questions above?
(d) What lessons can you draw from the above?

6. **Voting** In this course, we study several different voting methods, i.e., methods for converting lots of individual preferences into a group preference. But there are lots of methods that we will not cover. Pick at least two of them, and explain them in clear language that your classmates would understand. Compare and contrast them with the methods studied in class. Give examples of situations where these methods are actually used. Give examples to show why these methods might be desirable in some situations. Give other examples to show how these methods (like the ones studied in class) can yield perverse results.

Here are some voting methods not covered in class. For more information about these methods or to find others, see our textbook (in some cases) and do a web search. *If you choose this topic, also specify which methods you intend to cover.*

(a) The city of Cambridge, Massachusetts uses a voting method that is different from any we will study in this course. See

`http://www.ci.cambridge.ma.us/~Election/prop-voting.html`

(b) Do a web search to find out about *approval voting*, a particularly simple system.
(c) See the textbook [TA] and do a web search to learn about the *single transferable vote* (which is used to select the winners of the Academy Awards).
(d) Early in his administration, President Clinton nominated Prof. Lani Guinier to be Deputy Attorney General for Civil Rights. In the press, she was portrayed as a radical extremist and dubbed the "Quota Queen", so eventually her nomination had to be dropped. One of the ideas Guinier promoted was *cumulative voting.* (You'll find out more if you do a web search on her name.)

7. **Apportionment** The U. S. Constitution does not specify precisely how seats in the House of Representatives should be apportioned among the states. In this course, we study several apportionment methods. However, there are many other methods that we will not cover. Pick one or more of them, and explain them in clear language that your classmates would understand. Compare and contrast each method with those studied in class. Why was each method proposed? Give the historical context. Give examples of situations (if any) where these methods are actually used. For methods that have been used in this country, also describe legal challenges to them. Give examples to show how these methods (like all other apportionment methods) can lead to paradoxes or violate the Quota Rule.

For more information about the following methods, or to find others, see our textbook, other textbooks, and the web. *If you choose this topic, also specify which method(s) you intend to cover.*

(a) The Hill-Huntington method is what Congress presently uses.
(b) Condorcet proposed an apportionment method after the French revolution.
(c) South Carolina Congressman William Lowndes proposed a method in 1822, just before he was lost at sea.

8. **Proportional Representation** In most democratic countries, national elections work differently than they do here. Instead of voting for a person,

you vote for a party. Parliamentary seats are then apportioned among the political parties in rough proportion to how many votes they received, but the precise apportionment method used differs from country to country.

In some ways, proportional representation is more fair than our own system. In other ways, it is unfair (see our study of weighted majority voting).

Say what proportional representation is. Describe how (and how well) it works in various countries. Discuss strengths and weaknesses. Give examples of how this system can lead to perverse results.

9. **Electoral College** Write a history of the Electoral College. What was its original purpose? Describe all occasions where it has given us anomalous results. Give an example to show how it is possible for a candidate to win a majority of the electoral votes while garnering less than a quarter of the popular vote. (For simplicity, this example could involve a hypothetical country with fewer than 50 states.)

10. **Calendar.** In the western world, we presently use the Gregorian calendar for most purposes. It's pretty simple: there are 12 months in a year, the months have various lengths, and each year has 365 days, right? Not exactly: every year divisible by four has an extra day, right? Well, it's a little more complicated than that. Describe the situation more precisely.

There are many other calendars that have been or are being used here and elsewhere. Examples include the Julian, French Revolutionary, Islamic, Hebrew, and Mayan calendars. It's safe to say that any civilization that had sophisticated agriculture had a calendar of some sort.

Write a history of calendars. For each calendar you discuss, consider the following questions. How does it work? Who used it, and when? Why was it constructed in the way it was? If it was eventually dropped in favor of something else, why?

11. **Numeral systems** Other civilizations have denoted numbers differently from the way we do. For example, while we use Arabic numerals, the Romans used what are now known as Roman numerals. They would have denoted the number 78 by LXXVIII. The Maya and the Babylonians had systems that were not based on the numer ten.

Write a short history of number systems, comparing and contrasting examples from all over the world.

Reference: [If]

12. **Measurement** Consider lots of units of measurement like the foot, inch, minute, firkin, furlong, stone, meter, kilogram, light year, kalpa, pound, and ounce. Come up with your own list, including lots of units that we currently use, and some that we don't. For each unit you choose, conider the following questions. Who used it, and when? How and when was it standardized to be

what it currently is? How, if at all, is it related to other units? If it has been largely replaced, why?

13. **Reorganize your workplace.** (*See me before attempting this one.*) Do you presently have a job? If so, imagine that you have just been promoted by two levels. What would you do to make your workplace more efficient or effective? How much would your reform cost? How would you evaluate the success or failure of your reform? If it succeeds, how would you convince your bosses of this fact (so that they can promote you again)?

14. **Why computers cannot save us.** We will study the Travelling Salesman Problem in Chapter 6 of the textbook. The problem goes as follows. Suppose you need to visit every city on some list and then return home, thus travelling a circuit. In what order should you visit the cities in order to minimize your cost (or distance, or whatever).

One way to solve this problem is to consider every possible ordering of the cities, and compute the cost of each corresponding journey. This is known as the brute-force method, and it is covered in the book.

Suppose you want to solve Travelling Salesman Problem with 100 cities using the brute-force method. How many circuits must you investigate? (Feel free to round your numbers to one significant digit. For example, you can round 2985000 to 3000000.)

Suppose your computer can investigate one circuit per second. How many seconds are required? How many minutes? Hours? Days? Years?

Suppose you get a new computer that is 1000000 times faster than your old computer. How many years will it take your new computer to solve the problem?

Suppose you get a third computer that is again 1000000 times faster than your second. How many years now?

What is the lesson here?

15. **How should I have assigned these topics?** You will undoubtedly like some of these topics better than others. However, I don't want to have everyone writing on the same thing, so if your preferences match those of all of the other students in this class, then someone is going to be unhappy.

Here is the problem I face. Suppose I ask all students to tell me their preferences. How can I assign writing topics to the students in such a way that the overall satisfaction level of the class is approximately maximized, subject to the condition that there are never more than two people writing about the same topic?

Notice that I have not defined "overall satisfaction level". There are several different ways to do this.

Come up with at least two methods for assigning topics while keeping everyone as happy as possible under the circumstances. Test these methods in the real world. For example, you can ask your friends their preferences among several flavors of ice cream, perform an allocation using each of your methods, and see if everyone got something that he or she liked reasonably well.

Or perform a test that doesn't involve ice cream. In order to be meaningful, the test should involve the allocation of things that, like paper topics, are liked by some and not by others.

16. **Very Very Large Numbers.** The number *googol* is defined to be 10^{100} (or 1 followed by a hundred zeros). The number *googolplex* is defined to be 10^{googol}, which can also be written $10^{10^{100}}$, or 1 followed by a googol zeros. But don't bother to write it out in this last form; the mass of the ink required would be much greater than the estimated mass of the universe.

So, for practical purposes, googolplex is too large to be written as a decimal number. We can only write it down because we have exponential notation. Similarly, there are numbers out there which, for practical purposes, are too large to be written down in exponential notation. To write down such large numbers, we need an even more powerful notation.

Describe the notation given in Davis and Hersh, and give a few examples which show how powerful it is. Building on their notation, try to come up with a notation of your own which is even more powerful.

Reference: [DH, pp. 142– ?]

17. **Two-person, zero-sum games.** Suppose that two people, A and B, are playing a game where one person's loss is the other's gain. (Most typical games are like this. For example, if A wins a chess game, then B must lose it.) For that matter, what is a "best strategy"? Does each player have a best strategy?

The answers to these questions may involve something called "linear programming." But you don't have to know how linear programming works, just how it is used here.

References: [Br1], [Br2], [Br3], [BM], [Gu], [Dav], [Wi].

18. **The Prisoner's Dilemma.** Suppose that you and your partner in crime are both arrested. The district attorney can prove both of you guilty of a crime punishable by 3 years in jail. He believes that you are both also guilty of a more serious crime, but he won't be able to prove it unless someone confesses. So he offers each of you a deal: If one of you squeals and the other one doesn't, then the squealer gets only 1 year in jail (as a reward), and the other one gets the book thrown at him: 10 years. If both of you squeal, then you both get 5 years. If neither of you squeal, then you both get 3 years.

This is a situation which cannot be analyzed using zero-sum game theory, but which occurs often enough in real life (including business, politics, and biology). Describe the situation, describe how it arises, and describe some of

the attempts to analyze it.
References: [BM], [Br2], [Ho2], [Gu].

19. **Nim.** Nim is a simple game which arises in many contexts. Here are the rules: We begin with some pennies, arranged in several piles of various sizes. Two players take turns. A turn consists of removing any positive number of pennies from a single pile. There are two versions of this game.

(i) The object of the first game is to remove the last penny.
(ii) The object of the second game is *not* to remove the last penny.

The first game is analyzed in the reference below, which also points to other sources. Explain that analysis. The second game is harder. Tinker around to come up with, if not a winning strategy, then at least some rules of thumb.
Reference: [Be].

20. **What is calculus?** (You must understand what a function is in order to do this.) Why was calculus invented? What mathematical problems was it designed to solve? How is it applied in the real world?
References: [CR], [EB], [KR], [Saw], [Sp].

21. **Classical Paradoxes.** You have probably heard of paradoxes such as the sentence

This sentence is false.

The sentence can be neither true nor false, since if it is true, then it must be false, and if it is false, then it must be true. Then there is Zeno's Paradox, which seems to imply that motion is impossible. These and other paradoxes are given in Plato's *Parmenides*. Restate them in modern language. In particular, describe why they were considered paradoxical. Some of these paradoxes can now be resolved. Which ones? How? References: [Pl], [Q], [Ho2], [No].

22. **Infinity.** What is it? Are there several kinds of infinity, or are all infinities the same? References: [Dun], [Gu], [Ru2].

23. **Platonic Solids.** A Platonic (or "regular") solid is a polyhedron each of whose faces is a regular polygon. Two examples are the cube and the tetrahedron. Euclid showed that there are only five regular solids. Explain his proof in clear, modern language, using whatever illustrations you feel are necessary. Explain the cosmological significance of the regular solids to the ancient Greeks.
References: [E, Book 13], and any book on ancient cosmology.

24. **Higher Dimensions.** What are they? Do they have any relevance to real life? What are some higher-dimensional analogues of familiar geometric

objects? References: Too many to list, but you could try [Ru1].

25. **Irrationality of $\sqrt{2}$** We use numbers for counting things. Once you know how to count, it is natural to use numbers for comparison as well. Example: If you have 7 apples and I have 3 applies, then my supply of apples is $\frac{3}{7}$ the size of yours. The number $\frac{3}{7}$ can be used for comparing two collections, because it can be written as a fraction (or ratio) of whole numbers. We call such a number *rational.*

We also use numbers for measuring.

So we have two different kinds of numbers: numbers for length and numbers for measuring. Are they the same kind of numbers? Your intuition might suggest that there is only one kind of number, and the early ancient Greeks certainly believed this.

But your intuition would be wrong. Not all numbers are rational. Suppose you have a square whose sides have length 1 unit. If you believe the Pythagorean Theorem ("the only theorem that everyone has heard of"), then you have to believe that the diagonal of the square has length $\sqrt{2}$. That is, $\sqrt{2}$ is a measuring number. Is it rational?

The Pythagoreans discovered that $\sqrt{2}$ is *not* rational, a fact so explosive that they tried to keep it secret. (It is said that they killed the man who blabbed.)

When someone makes a claim like "$\sqrt{2}$ is not rational," the following natural question should arise in your head: "How can *anyone* possibly know such a thing?" I am claiming that there is no way to write $\sqrt{2}$ as a fraction of whole numbers. I am not just claiming that there is no *known* way to write $\sqrt{2}$ as a fraction; I am claiming that there *is no way.*

Thus, you should be skeptical of the claim. Do I make the claim because I tried really hard to write $\sqrt{2}$ as a fraction, failed, and assumed that therefore no one else could do it either?

On the flip side, if you ever come to understand how we can know for sure that $\sqrt{2}$ is irrational, you should be surprised at how much you have learned.

Look up the reason how we know that $\sqrt{2}$ is irrational. Write it up in a way that your classmates would find convincing. Include some historical context.

26. **Simpson's Paradox.** Suppose you are having surgery, and can choose between Hospital A and Hospital B. Which is safer?

	A	B
Death Rate	3 %	2 %
Death Rate for sick patients	3.8%	4 %
Death Rate for healthy patients	1 %	1.3%

According to the table, A has a higher death rate, so B is safer. But wait. There are two kinds of patients: sick and healthy. For both kinds of patients, A is safer! This is an example of Simpson's Paradox.

The paradox has nothing in particular to do with hospitals. Sometimes it is relevant to the search for hidden bigotry in sentencing, or the ranking of baseball players, etc. See our textbook (Chapter 14, Exercise 77) for an explicit, numerical example. For others, as well as help in resolving the paradox, see any statistics textbook or do a web search. (On the web, there's even a page that helps you to construct examples of your own.)

27. **Surveying** How is it possible to measure the heights of mountains? How is it possible to redraw property lines without reference to any markers on the ground? (The ancient Egyptians had to do this every year, after the Nile flood washed away all of the markers.) How do we know exactly where international and state borders are? What other things do surveyers do?

28. **MAKE MONEY FAST!** Have you ever received a chain letter or e-mail that promised that you'd make lots of money if you sent a little money to a few strangers? If so, then someone was trying to recruit you into a *pyramid scheme*. I get these solicitations often, and an example is included below.

Pyramid schemes are illegal in all fifty states and most foreign countries. (Go to the web site of the U. S. Postal Service to find the relevant laws.) The purpose of the present exercise is to see one of several reasons why this is the way it should be.

Let's suppose that you start a pyramid such as the one described in the attached letter. By following the recommended procedures, you manage to persuade 16 people to send you a dollar each.

There are now 17 people in the scheme, including you. You are at level 0 of the pyramid, and the other 16 are at level 1. So far, what percentage of the people in this scheme have lost money?

The 16 people at level 1 of the pyramid then recruit another 16 people each, or 256 people in all. They make up level 2. The people on level 2 send a dollar each to the people above them in the pyramid. So far, what percentage of the people in this scheme have lost money?

The 256 people on level 2 then recruit 16 people each into level three, for a total of $16 \times 256 = 4096$ people. So far, what percentage of the people in this scheme have lost money?

Consider the next natural question in this series of questions.

Consider the *next* natural question in this series of questions.

If each partipant, following the advice in the letter, sends bulk e-mail to 100,000 people, then how many pieces of e-mail have been generated by this scheme so far?

Can the scheme continue forever? If the scheme ends, approximately what percentage of its participants will have lost money?

Some variants of the pyramid scheme chain letter claim that if we just keep the chain going, perhaps partipating several times, *everyone* can get rich. Is this so?

Here is a letter I received recently.

```
Dear Friend,

This letter is about an opportunity to make an incredible amount of
Money ( CASH !!!) in a very short time. The cost is only $6.00! This is the
16th day since I started receiving $ cash, and so far I have received $5,845
(in $1 Bills)...so I guess this is really working! Give it a try! All I did
was follow the instructions in the letter that I received below, and sent out
some e-mail to people who responded to my ads.

Here is a testimony from one of the thousands who have benefited from
this simple investment plan.

"I'm a retired attorney, and about a year ago a man came to me with a letter.
The letter he brought to me is the same letter before you now. He asked me to
verify that this letter was legal. I told him that I would review it and get
back to him.  When I first read the letter, I thought it was some off the wall
idea to make money.  A week later I met again with my client to discuss the
issue. I told him that the letter will be all right. I was curious about the
letter, so he told me how it worked. I thought it was a long shot, so I
decided against participating.  Before my client left, I asked him to keep me
updated as to his results. About two months later he called me to tell me that
he had received more than $800,000.00 in cash! I didn't believe him so
he asked me to try the plan and see for myself."

"I thought about it for a few days and decided that there was not much to lose.
I followed the instructions exactly and mailed out 200 letters. Sure enough the
money started coming in! It came slowly at first, but after three weeks I was
getting more than I could open in a day. After three months the money stopped
coming. I kept a precise record of my earnings and at the end it totaled
$868,439.00. I earn a good living as an attorney, but as anyone in the legal
profession will tell you, there is a lot of stress that comes with the
territory. I decided if things worked out, I would retire from practice and
play golf. This time I sent out 500 letters. Well, three months later, I had
totaled $2,344,178.00."

"I met my old client for lunch to find out exactly how it works. He told me
that there were a few similar letters going around. What made this one
different is the fact that there were six names on the letter, not three like
most others. That act alone resulted in more returns. The other factor was the
advice I gave him in making sure the whole thing was perfectly legal, since no
one wants to risk doing anything illegal. I bet now you are curious about what
little changes I told him to make. Well, if you send a letter like this one
out, to be legal, you must sell something if you expect to received a dollar.
I told him that anyone sending a dollar must received something in RETURN.
So when you send a dollar to each of the six names on the list, you must
include a slip of paper saying, "Please add me to your mailing list" and
include your name and mailing address. This is the key to the program. The
item you will received for your dollar sent, is THIS letter and the right to
earn thousands."

Follow the simple instructions EXACTLY, and in less than three months
you should receive MORE THAN $800,000.00 IN COLD HARD CASH!

1) IMMEDIATELY send $1.00 (US$) to each of the six people listed below.
THE SOONER YOU SEND THE "$1.00 LETTERS" THE SOONER YOU CAN
START GETTING A RETURN! Wrap the dollar in a note saying
"PLEASE ADD ME TO YOUR MAILING LIST".


1. T. Vandegrift 398 West St., Louisville, CO 80027

2. Janette Cheramie 176 East 25th Place Cut Off, LA 70345

3. E. McDonald, 6006 Blue Ridge Dr. #D, Highlands Ranch, CO 80126
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4. M Collie 4612 Maryvale Dr. NE, Calgary, AB T2A 2T1 Canada

5. BRM 514 Lowell Ave #1 Cincinnati, OH 45220

6. T. Beck 103 W. Franklin St. Topton, PA 19562

2) REMOVE the NAME and ADDRESS NEXT to #1 at the top of the list and move the rest of the names UP one position. Then place YOUR name in the #6 spot. This is best done by saving this to a file and entering your information on line #6. Be careful when you type the addresses. Don't forget to PROOFREAD them. Make SURE that the names and addresses are correct.

3) When you have COMPLETED the above instructions, you have several options on how you market the letter - through the Postal Service, through E-mail, through posting in "FREE CLASSIFIED ADS" and "NEWSGROUPS" ON THE INTERNET, or through whatever way you think is most effective.

To send this letter out to thousands of people and increase your profits, I suggest you use a Bulk E-mail company. Call 207-896-7915 to have your letter emailed. They are fast, effective and give excellent service. 100,000 emailings costs just $89.00 and they are running a special now of 50,000 additional mailings FREE!!!

This letter has been proven perfectly legal for all of the above as long as you follow the instructions, because you are purchasing membership in our exclusive mailing list. The more you send out, the more YOU will make. We strongly encourage you to mail this letter to family, friends, and relatives as well.

THIS IS A SERVICE AND IS 100% LEGAL. (Refer to title 18, section 1302 &1341 of the US Postal and Lottery Laws)

Assume for example you get a 8% return rate.

1) When you mail out 200 letters, ONLY 16 people send you $1.00

2) Those 16 people mail out 200 letters, (3200 letters) and ONLY 256 people send you $1.00

3) Those 256 people mail out 200 letters, (51,200 letters) and ONLY 4, 096 people send you $1.00

4) Those 4,096 people mail out 200 letters (812,200 letter) and ONLY 65,536 people send you $1.00

5) Those 69,536 people mail out 200 letters (13,107,200 letters) and ONLY 1,048,576 people send you $1.00. At the Next level your name-drops off the list.

Think about it. Look what you WILL have BEFORE your name-drops off the list! I know this looks and sounds unbelievable. Just try it, and you will be happy that you did because you will received proof when THOUSANDS of ONE-DOLLAR BILLS start to pile up!

***MAKE SURE you send One US dollar to each of the six names on the list. (This is VERY IMPORTANT), with a note saying "PLEASE ADD ME TO YOUR MAILING LIST"

P.S. You've read this far, so let me as you one simple question,

WHAT HAVE YOU GOT TO LOSE?

Even with a 1% return you will still get $100,000.00 in 90 DAYS!!! What you can gain is an income, like the example in this letter. You will have a very small expense, but you will reap HUGE potential returns. What do you have to lose? I invite you to JOIN our mailing list RIGHT NOW!

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Thanks you for your time. GOOD LUCK & BEST WISHES TO YOU !!!
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Remember: the above scheme is illegal, despite the letter's claims to the contrary.

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[Ru1] Rudy Rucker, *Geometry, Relativity, and the Fourth Dimension.*

[Ru2] Rudy Rucker, *Infinity and the Mind.*

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[Sp] Michael Spivak, *Calculus.*

[TA] Tannenbaum and Arnold, *Excursions in Modern Mathematics.*

[Wi] J. D. Williams, *The Compleat Strategyst.*