

There are many situations in which we must divide a set of goods (candy, money, property, etc.) among a collection of players (people, businesses, states, etc.) fairly.

1. Important ingredient: Each player must have a *value system*—the ability to assign value to the set and to any part of it.

Each player may have a different value system.

Example. Alice, Bob, Carol, and Ted go out for a Chinese dinner, and each orders a main dish:

A	Steamed Vegetables
B	Sweet and Sour Pork
C	Szechuan Beef
T	Garlic Chicken

Alice is a vegetarian, except for chicken. Bob is allergic to garlic. Carol doesn't eat pork. Apart from that, everyone likes everything equally. With this in mind, how does each value the food that will be served (as a percentage or fraction of the total value of the meal)?

	SV	S+SP	SzB	GC
A	50%	0%	0%	50%
B				
C				
T				

2. **Definition:** If there are N players, then a *fair share* will mean any share that, *in the opinion of the player receiving it*, has a value at least one N^{th} of the total value.

Example. Now Alice, Bob, Carol, and Ted want to divide a cake among themselves. Alice cuts the cake into four slices (s_1, s_2, s_3, s_4). The table below gives the value of each piece in the eyes of each player. For each player, indicate which of the slices are fair shares.

	s_1	s_2	s_3	s_4	Whole cake
A	\$4.50	\$4.50	\$4.50	\$4.50	\$18
B	\$2.75	\$2.25	\$2.50	\$2.50	\$10
C	\$4.00	\$3.50	\$1.50	\$3.00	\$12
T	\$3.00	\$5.00	\$5.00	\$2.00	\$15

3. **Divider-Chooser method** for two players: “I cut; you choose.”
4. **Lone-Divider method** for more than two players:
 - (a) Divide: Divider cuts cake into what he/she thinks are equal shares.
 - (b) Bid: Each chooser writes down privately which piece(s) he/she thinks are fair.
 - (c) Distribute: Classify the “bid-for” and “unbid” pieces.
 - i. If 1 or 0 pieces are unbid pieces, each person gets a piece he/she thinks is fair.
 - ii. If there are 2 or more unbid pieces, the distribution depends on the number of players and the number of pieces in the unbid set.
5. **Last-Diminisher method:** N players
 - (a) P_1 cuts off a slice s he/she considers fair.
 - (b) If P_2 thinks the slice P_1 cut off was fair, he/she simply passes. On the other hand if P_2 thinks the slice s is more than fair, he/she trims off a piece of s , returns the trimming to the center, and claims the remainder of the slice s for himself/herself.
 - (c) The rest of the players get the same chance to either pass OR trim and claim.
 - (d) The last person to trim the piece gets to keep it and drop out.
 - (e) Start over with the remaining players.