

1. **Apportionment problem:** Fair division problem in which players are entitled to different-sized shares.
2. **Standard divisor (SD):** total number of people to be served, divided by total number of servers (The standard divisor gives the number of people served by each server, on average.)
3. **Standard quota (SQ):** number of people to be served in a subgroup, divided by the standard divisor (The standard quota gives the number of servers per subgroup, allowing for fractional servers.)
4. A clinic has a nursing staff of 225 nurses working in four shifts: *A* (7am–1pm), *B* (1pm–7pm), *C* (7pm–1am), and *D* (1am–7am).

The number of nurses apportioned to each shift is based on the average number of patients per shift, given in the following table:

Shift	Avg. # patients	Standard quota	LQ	UQ	RQ
A	871				
B	1029				
C	610				
D	190				
Total	2700				

5. **Lower quota (LQ):** first counting number smaller than the standard quota
6. **Upper quota (UQ):** first counting number larger than the standard quota
7. **Rounded quota (RQ):** rounded-off standard quota (Round down if the fractional part is less than 0.5; round up if the fractional part is greater than or equal to 0.5.)

8. Alexander Hamilton's method for apportionment:
- Calculate standard quotas.
 - Give each subgroup its lower quota.
 - Give the surplus (leftover) servers one at a time to the subgroups whose standard quotas have the largest fractional parts.
9. Apply Hamilton's method to find the final apportionment (app't) of the 225 nurses.

Shift	Avg. # patients	SQ	LQ	Surplus	App't
A	871	72.58	72		
B	1029	85.75	85		
C	610	50.83	50		
D	190	15.83	15		
Total	2700	224.99	222		

10. A university has 10,020 students enrolled in math classes, 9,030 in statistics (stats) classes, and 950 in computer science (CS) classes. How would the dean apportion 200 faculty positions among the three disciplines using Hamilton's method?

Subject	Students	Standard quota	LQ	Surplus	App't
Math	10,020				
Stats	9,030				
CS	950				
Total	20,000				

11. The dean finds out that there is an extra faculty member available. How would he apportion 201 faculty positions (instead of 200) among the three disciplines using Hamilton's method?

Subject	Students	Standard quota	LQ	Surplus	App't
Math	10,020				
Stats	9,030				
CS	950				
Total	20,000				

What seems unfair?

12. Problems with Hamilton's method

- (a) **Alabama paradox:** An increase in the total number of servers can force a subgroup to lose one of its servers.
- (b) **Population paradox:** Subgroup X has a population growth rate higher than that of subgroup Y. When the apportionment is recalculated based on the new population figures, X loses a server to Y.
- (c) **New-states paradox:** The addition of a new subgroup with its fair share of servers can affect the apportionment to other subgroups (*e.g.* states).