

We are trying to divide a set S of goods (pizza, land, property, candy etc.) among a collection of players (people, businesses, states, etc.) fairly.

1. Goods can be

- (a) **continuous**: divisible infinitely many ways,
- (b) **discrete**: distributable as indivisible objects,
- (c) **mixed**: some of each.

2. **Method of markers**: N players, M items

- * Want at least several times as many items as players.
- * Items should be small and of homogeneous (not radically different) value.

- (a) **Arrangement**: Place the M items in a line (array).
- (b) **Bids**: Each player divides the array into N acceptable segments by writing down privately the locations of his or her $N - 1$ markers.
- (c) **Allocations**: Place all the players' markers in the array.
Scan the array from left to right until the first *first marker* is located. The player owning that marker gets to keep his or her first segment, and that player's markers are removed. (Break ties by drawing lots.) Continue moving from left to right looking for the first *second marker*. The player owning it gets to keep EXACTLY his or her second segment (no more). That player's markers are removed.
Continue the process until each player has received one of his or her segments.
- (d) **Leftovers**: There are no leftovers at the ENDS of the originally array, but there are usually leftovers in the middle. Distribute them by trying to give everybody a fair share. This is straightforward for a small number of leftovers. If that doesn't work, draw names. If there are a lot of leftovers, we could do recursion: reapply the method of markers on the leftovers.

(OVER)

3. Method of sealed bids: N players, M items

- * Works best if the number of items and players is somewhat close.
 - * The items can be of significantly different value.
- (a) Bids: Each player bids on each item by writing down privately how much he or she thinks each item is worth.
 - (b) Allocations: Compile the results of the bids (say, into a table). Each item goes to the player who bid the most for it.
 - (c) Payments: In each player's value system, calculate how much is exactly a fair share. (Total up the player's bids and divide by THE NUMBER OF PLAYERS.)
For each player, check whether the property he or she received is more or less than his or her fair share. If it is MORE, the player must put the difference into "the pot." If it is LESS, the player is owed the difference *from* the pot. Therefore, collect payments to the pot, and distribute payments from the pot.
 - (d) Surplus: Divide what's left in the pot equally among the N players.