

10. Calculate  $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 4x}}{4x + 1} \cdot \frac{1/x}{1/x}$ .

13 pts

$$= \lim_{x \rightarrow -\infty} \frac{-\sqrt{\frac{x^2 + 4x}{x^2}}}{\frac{4x}{x} + \frac{1}{x}} \quad x = -\sqrt{x^2} \text{ for } x < 0$$

$$= \lim_{x \rightarrow -\infty} \frac{-\sqrt{1 + \frac{4}{x}}}{4 + \frac{1}{x}} = -\frac{1}{4}$$

11. **Extra credit:** If  $f(1) = 10$ , and  $f'(x) \geq 2$  for  $1 \leq x \leq 4$ , how small can  $f(4)$  possibly be? <sup>\*</sup>  
When you apply a theorem, refer to it by name, and show that all the hypotheses of the theorem are satisfied. State your final answer in a complete sentence.

$f(x)$  is cont on  $[1, 4]$ , (1)

&  $f(x)$  is diff on  $(1, 4)$ , (2).

So by the MVT,

$$f'(c) = \frac{f(4) - f(1)}{4 - 1}$$

$$f'(c) \geq 2 \Rightarrow \frac{f(4) - f(1)}{3} \geq 2$$

$$f(4) \geq 6 + 10$$

The largest  $f(4)$  can be is 16.

5 pts

\* assuming (1) & (2).