

5. Find the linearization of the function $f(x) = \frac{1}{\sqrt{2+x}}$ at $a = 0$.

$$L(x) = f(0) + f'(0)(x-0)$$

4 pts

$$f'(x) = -\frac{1}{2} \frac{1}{(\sqrt{2+x})^3}$$

$$= \frac{1}{\sqrt{2}} - \frac{1}{2} \frac{1}{2\sqrt{2}} x$$

(Recall the graph of $y = L(x)$ is the tangent line to the graph of $y = f(x)$ at $x = 0$.

6. Consider the function $y = 16/x$. \uparrow The line approximates the graph of $y = f(x)$ well for x near 0.

- (a) If $x = 4$ and the change in x (Δx) is -1 , what is the corresponding change in y (Δy)?

$$\Delta y = f(x + \Delta x) - f(x)$$

2 pts

$$= f(3) - f(4) = \frac{16}{3} - 4 = \frac{16-12}{3} = \frac{4}{3}$$

- (b) Give a geometric interpretation of the ratio $\Delta y / \Delta x$ for this problem. Your answer must be a single complete sentence.

2 pts

$\frac{\Delta y}{\Delta x}$ is the slope of the secant line to the graph of $y = f(x)$ through the points $(4, f(4))$ & $(3, f(3))$.

- (c) If $x = 4$ and the change in x (dx) is -1 , what is the corresponding differential dy ?

2 pts

$$dy = f'(x) dx = f'(4)(-1) = 1$$

$$f'(x) = -\frac{16}{x^2}$$

- (d) Give a geometric interpretation of the ratio dy/dx for this problem. Your answer must be a single complete sentence.

2 pts

dy/dx is the slope of the tangent line to the graph of $y = f(x)$ through the point $(4, f(4))$.