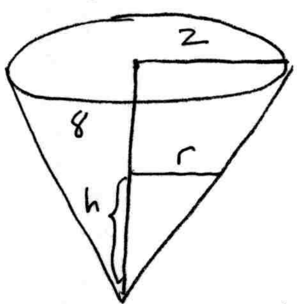
3. A water tank has the shape of an inverted circular cone with base radius 2 m and height 8 m. If water is being pumped into the tank at a rate of 2 m^3/min, find the rate at which the water level is rising when the water is 5 m deep. (Recall that the volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$.)

13 pts



$V = \frac{1}{3}\pi r^2 h$

$\frac{dV}{dt} = 2\,m^3/min$ $\quad \frac{dh}{dt} = ?$

$h = 5\,m.$

$\frac{8}{2} = \frac{h}{r} \Rightarrow r = \frac{h}{4}$

$V = \frac{1}{3}\pi\left(\frac{h}{4}\right)^2 \cdot h = \frac{1}{3\cdot 16}\pi h^3 \Rightarrow \frac{dV}{dt} = \frac{\pi}{\cancel{3}\cdot 16}\cdot\cancel{3}h^2\frac{dh}{dt}$

$2 = \frac{\pi}{16}\cdot 25\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{32}{25\pi}\ m/s.$

$\approx .407\ m/s.$

4. Verify that $f(x) = x^2 - 4x + 1$ satisfies the three hypotheses of Rolle's Theorem on $[0, 4]$. Then find all the numbers c that satisfy the conclusion of Rolle's Theorem.

13 pts

1 $f(x)$ is cont on $[0,4]$ because a polynomial must be cont on $\mathbb{R}$.

2 $f(x)$ is diff on $[0,4]$ because a polynomial must be diff. on $\mathbb{R}$.

3 $f(0) = f(4) = 1.$

$f'(c) = 0 \Leftrightarrow 2c - 4 = 0 \Leftrightarrow \underline{c = 2}.$

$0 < 2 < 4$ ✓