

8. Evaluate $\lim_{x \rightarrow 0} \frac{2 - \sqrt{4 - x^2}}{x}$, or state why the limit does not exist.

10 pts
10

$$\lim_{x \rightarrow 0} \frac{2 - \sqrt{4 - x^2}}{x} \cdot \frac{2 + \sqrt{4 - x^2}}{2 + \sqrt{4 - x^2}} = \frac{4 - (4 - x^2)}{x(2 + \sqrt{4 - x^2})}$$

$$= \frac{x^2}{x(2 + \sqrt{4 - x^2})} = \frac{x}{2 + \sqrt{4 - x^2}} \quad \checkmark \quad \lim_{x \rightarrow 0} \frac{2 - \sqrt{4 - x^2}}{x} = 0$$

9. Consider the function $\frac{2t+1}{t+3}$.

14 pts
14

(a) Find $f'(a)$. (Show the use of limits in calculating your answer.)

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{2(a+h)+1}{(a+h)+3} - \frac{2a+1}{a+3}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{2a+2h+1}{a+h+3} - \frac{2a+1}{a+3}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{(2a+2h+1)(a+3) - (2a+1)(a+h+3)}{(a+h+3)(a+3)}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2a^2 + 6a + 2ah + 6h + a + 3 - 2a^2 - 2ah - 6a - a - h - 3}{h(a+h+3)(a+3)}$$

$$= \lim_{h \rightarrow 0} \frac{5h}{h(a+h+3)(a+3)}$$

$$f'(a) = \frac{5}{(a+3)^2} \quad \checkmark$$

(b) Find the equation of the tangent line to the curve $y = f(t)$ at the point $(-1, -1/2)$. (There is no need to simplify your answer.)

$$y = m(x) + b$$

$$-1/2 = \frac{5}{4}(-1) + b$$

$$b = 3/4$$

$$y = \frac{5}{4}x + \frac{3}{4} \quad \checkmark$$