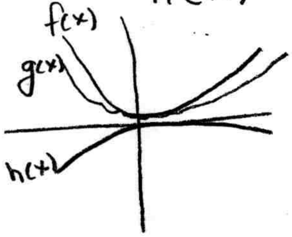
5. Use the Squeeze Theorem to prove that $\lim_{x \to 0} x^4 \cos\left(\frac{2}{x}\right) = 0$.

4 pts

$f(x) = x^4$

$g(x) = x^4 \cos(\frac{2}{x})$

$h(x) = -x^4$



$\lim_{x \to 0} f(x) = \lim_{x \to 0} x^4 = 0$

$\lim_{x \to 0} h(x) = \lim_{x \to 0} -x^4 = 0$

g(x) is between f(x) and h(x). Therefore by the Squeeze thm $\lim_{x \to 0} g(x) = 0$ ✓

6. Find the largest possible value of δ so that

$$|\sqrt{4x+1} - 3| < 0.1 \text{ whenever } 0 < |x-2| < \delta.$$

16 pts

$2.9 < \sqrt{4x+1} < 3.1$

$2.9^2 < 4x+1 < 3.1^2$

$8.41 - 1 < 4x < 8.61$

$1.853 < x < 2.153$

$\delta = |1.853 - 2| = .147$

$\delta = |2.153 - 2| = .153$

$\delta = .147$

7. Use the definition of continuity and the properties of limits to show that $f(x) = x^3 + \sqrt{6-x}$ is continuous at $x = 2$.

8 pts

$\lim_{x \to a} x^3 + \sqrt{6-x}$

$= \lim_{x \to a} x^3 + \lim_{x \to a} \sqrt{6-x}$

$= \lim_{x \to a} x^3 + \sqrt{\lim_{x \to a} 6 - \lim_{x \to a} x}$

$= a^3 + \sqrt{6-a} = f(a).$

for all a in the domain of f(x), including a=2

✓ 1. f(x) is defined at 2

✓ 2. $\lim_{x \to 2} f(x)$ exists.

✓ 3. $\lim_{x \to 2} f(x) = f(2)$

(OVER)

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