

Laura Gross

3. Evaluate $\lim_{x \rightarrow 0} \frac{1 - \sqrt{1-x^2}}{x}$, or state why the limit does not exist.

8 pts

$$\frac{(1 - \sqrt{1-x^2})(1 + \sqrt{1-x^2})}{x(1 + \sqrt{1-x^2})} = \frac{1 - (1-x^2)}{x(1 + \sqrt{1-x^2})}$$
$$= \frac{x^2}{x(1 + \sqrt{1-x^2})} = \frac{\overset{\circlearrowleft}{x} \rightarrow 0}{1 + \sqrt{1 - \underset{\circlearrowright}{x^2} \rightarrow 0}} \rightarrow \boxed{0} \quad 1-x^2 \geq 0$$

as $x \rightarrow 0$

4. Use the Intermediate Value Theorem to show that the equation $\sin(x) = 3 - 2x$ has a solution on the interval $(0, \pi/2)$. Remember to show the hypotheses of the theorem are satisfied.

6 pts

$f(x) = \sin x - 3 + 2x$ is cont. on $[0, \pi/2]$ ✓

$$f(0) = -3$$

$$f(\pi/2) = 1 - 3 + \pi > 0$$

$$f(0) < 0 < f(\pi/2)$$

$\therefore f(c) = 0$ for some c in $(0, \pi/2)$ ✓