

Name: _____

1. Suppose $f(x)$ is a one-to-one function, $f(-1) = 2$, $f'(-1) = 3$, and $g = f^{-1}$.

10 pts

(a) State what the value of $g'(-1)$ MUST be, or state that it “cannot be determined” with the information given.

(b) State what the value of $g'(2)$ MUST be, or state that it “cannot be determined” with the information given.

2. Show the function $y = e^x + e^{-x/2}$ satisfies the differential equation $2y'' - y' - y = 0$.

10 pts

3. Suppose $f(x) = \frac{x}{1 - \ln(x - 1)}$.

10 pts

(a) What is the EXACT domain of $f(x)$? (Do not use decimal approximations in your answer.)

(b) Calculate $f'(x)$.

(OVER)

4. Find an expression for dy/dx if $x^y = y^x$.

10 pts

5. Calculate $\int_e^6 \frac{dx}{x \ln x}$ EXACTLY. Show your substitutions, and do not use decimal approximations in your answer.

9 pts

6. Calculate $\lim_{x \rightarrow 0^+} \tan^{-1}(\ln(x))$ EXACTLY if possible, or state that the limit does not exist. Show your reasoning without using decimal approximations.

6 pts

7. USE IMPLICIT DIFFERENTIATION to calculate $\frac{d}{dt} \cos^{-1}(t)$.

8 pts

8. Calculate $\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} dx$. Show your substitutions.

8 pts

9. Calculate $\int \frac{dt}{\sqrt{9t^2-1}}$. Show your substitutions.

8 pts

10. USE THE DEFINITION of $\sinh(x)$ to calculate $\frac{d}{dx} \sinh(x)$.

6 pts

11. Calculate the limits if possible, or state the limit does not exist. USE EXACT METHODS.

15 pts

(a) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{1 - x}$

(b) $\lim_{x \rightarrow 0} \frac{\cos x - 1}{3x^2}$