T4, Sp2007 p1.                                        Laura Kross

1. Consider the SEQUENCE $a_n = \dfrac{(-1)^{n-1} n}{n^2+1}$. (NOTE THIS PROBLEM HAS NOTHING TO DO WITH SERIES. It deals with a SEQUENCE.)

§12.1 HW(21)

(a) State the limit of the SEQUENCE, or state that the sequence diverges.

| 4 pts |
|---|
| |

$a_n \to \boxed{0}$

like §12.1 54-60

(b) The SEQUENCE is (circle one): INCREASING DECREASING NOT MONOTONIC

| 4 pts |
|---|
| |

see chapter review T-F quiz

2. For each statement about infinite series, circle TRUE or FALSE. If it is false, GIVE AN EXAMPLE THAT DISPROVES THE STATEMENT.

(a) If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ must converge. TRUE FALSE

| 4 pts |
|---|
| |

If false, give an example that disproves the statement.

$\sum_{1}^{\infty} \frac{1}{n}$

(b) If $\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ must diverge. TRUE FALSE

| 4 pts |
|---|
| |

If false, give an example that disproves the statement.

(c) If $a_n > 0$, and $\sum a_n$ converges, then $\sum(-1)^n a_n$ must converge. TRUE FALSE

| 4 pts |
|---|
| |

If false, give an example that disproves the statement.

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§12.6(17) HW3

3. Show whether the series $\sum_{n=1}^{\infty}(-1)^n \frac{n}{5+n}$ is (circle one)

**divergent** **absolutely convergent,** or **conditionally convergent.**

| 10 pts |
|---|
| |

$$\lim_{n\to\infty} \frac{n}{5+n} = 1$$

So $\lim_{n\to\infty} (-1)^n \frac{n}{5+n}$ does not exist.

So $\lim_{n\to\infty} (-1)^n \frac{n}{5+n} \neq 1$

So series diverges by the divergenie test.

Chapter review (27)

4. Find the sum of $\sum_{n=1}^{\infty} \frac{2^{2n+1}}{5^n}$, or state that the series diverges.

| 10 pts |
|---|
| |

$$\sum_1^{\infty} \frac{2^{2n}2}{5^n} = \sum_1^{\infty} \frac{(2^2)^n 2}{5^n} = \sum_1^{\infty} \frac{4^n 2}{5^n} =$$

$$= \sum_1^{\infty} 2\left(\frac{4}{5}\right)^n = \frac{2(4/5)}{1-4/5} = 2\frac{4}{\cancel{5}} \cdot \frac{\cancel{5}}{1} = \boxed{8}$$

Series is geometric with $|r| = \frac{4}{5} < 1$,

So it converges to $\frac{a}{1-r}$.

**(OVER)**

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chapter review (15)

5. Show whether the series $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln(n)}}$ is (circle one)

**divergent** **absolutely convergent,** or **conditionally convergent.**

| 10 pts |
|---|
| |

$f(x) = \frac{1}{x\sqrt{\ln x}}$ is continuous, positive, decreasing on $[2,\infty)$ & $f(n) = a_n = \frac{1}{n\sqrt{\ln n}}$.

$\int_2^{\infty} \frac{1}{x\sqrt{\ln x}}\,dx \qquad u = \ln x \rightarrow du = \frac{1}{x}\,dx$.

$= \int_{\ln 2}^{\infty} u^{-1/2}\,du = \lim_{t\to\infty} 2u^{1/2}\Big|_{\ln 2}^{t} = \infty$.

series diverges by the integral test

likes §12.7 (30)

6. Show whether the series $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+5}$ is (circle one)

**divergent** **absolutely convergent,** or **conditionally convergent.**

| 10 pts |
|---|
| |

$a_n = \frac{\sqrt{n}}{n+5} > 0, \quad b_n = \frac{1}{\sqrt{n}} > 0.$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sqrt{n}}{n+5}\,\frac{\sqrt{n}}{1} = 1$, which is positive & finite.

$\sum b_n$ diverges: It's a p-series w/ $p = \frac{1}{2} \le 1$.

So $\sum a_n$ diverges by limit comparison test.

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§12.6 (20) ~~7. Show~~ whether the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^n}$ is (circle one)

**divergent** **absolutely convergent,** **or conditionally convergent.**

| 10 pts |
|---|
| |

$$|a^n|^{1/n} = \left[\frac{1}{(\ln n)^n}\right]^{1/n} \qquad n = 2, 3, \ldots$$

$$= \frac{1^{1/n}}{(\ln n)^{n \cdot 1/n}} = \frac{1}{\ln n} \to 0 \text{ as } n \to \infty.$$

$0 < 1 \Rightarrow$ series converges absolutely by the root test.

chapter review (42) ~~8. Find~~ the radius of convergence $R$ of the power series $\sum_{n=1}^{\infty} \frac{2^n (x-2)^n}{(n+2)!}$, and **put a box around your answer.**

| 10 pts |
|---|
| |

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{2^{\cancel{n+1}\,1}(x-2)^{\cancel{n+1}\,1}}{(n+3)!} \cdot \frac{(n+2)!}{\cancel{2^n}\cancel{(x-2)^n}}\right| =$$

$$= \left|\frac{2(x-2)}{(n+3)\cancel{(n+2)!}} \frac{\cancel{(n+2)!}}{1}\right| \to 0 \text{ as } n \to \infty.$$

$0 < 1$ for all $x$.

So series converges absolutely by the ratio test for all $x$

$\boxed{R = \infty}$

(OVER)

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class notes: §12.4 (34)

9. Consider the convergent series $\sum_{n=1}^{\infty} \frac{1+\cos(n)}{n^5}$. Estimate the error in using $s_{10}$ as an approximation for the sum of the series. Use the hint that $\frac{1+\cos(n)}{n^5} < \frac{2}{n^5}$ for $n = 1, 2, 3, \ldots$.

10 pts

$$R_{10} \leq \int_{10}^{\infty} \frac{2}{x^5}\,dx$$

$f(x) = \frac{2}{x^5}$ is continuous, positive, decr.

$$= \lim_{t\to\infty} \frac{2x^{-4}}{-4}\Big|_{10}^{t} = \lim_{t\to\infty} \left(-\frac{1}{2}\right)\left[\frac{1}{t^4} - \frac{1}{10^4}\right]$$

$$= \frac{1}{2} 10^{-4} = \frac{1}{2}(.0001) = 0.00005$$

$$R_{10} \leq 0,00005$$

class notes: part of §12.8 (10)

10. Show whether the series $\sum_{n=2}^{\infty} \frac{(-3)^n}{n3^n}$ is (circle one)

divergent, absolutely convergent, or conditionally convergent

10 pts

$$\sum_{n=2}^{\infty} \frac{(-1)^n \cancel{3^n}}{n \cancel{3^n}} \qquad b_n = \frac{1}{n} > 0.$$

(i) $\lim_{n\to\infty} \frac{1}{n} = 0.$

(ii) $\frac{1}{n+1} < \frac{1}{n}$ ✓ $n = 2, 3, \ldots$

$\Leftrightarrow b_{n+1} \leq b_n$

Series converges by the alternating series test

But $\sum |a_n|$ diverges because it is the harmonic series.