Laura Gross

1. Set up but do not evaluate an integral for the length of the curve $y = \cos(x)$, $0 \leq x \leq 2\pi$.

10 pts

$y = f(x) = \cos x.$

$$\int_0^{2\pi} \sqrt{1 + [f'(x)]^2}\, dx.$$

$$= \int_0^{2\pi} \sqrt{1 + (-\sin x)^2}\, dx.$$

2. Set up but do not evaluate an integral for the area of the surface obtained by rotating the curve $x = \frac{1}{3}(y^2+2)^{3/2}$, $1 \leq y \leq 2$, about the x-axis.

10 pts

$\int 2\pi r\, ds \qquad x = g(y) = \frac{1}{3}(y^2+2)^{3/2}.$

$$\int_1^2 2\pi y \sqrt{1 + [g'(y)]^2}\, dy.$$

$$= \int_1^2 2\pi y \sqrt{1 + \left[\frac{1}{3}\cdot\frac{3}{2}(y^2+2)^{1/2}(2y)\right]^2}\, dy.$$

3. Find parametric equations for the path of a particle that moves along the circle $x^2 + y^2 = 9$ in the manner described. (Remember to give an interval for the parameter.)

10 pts

(a) Twice around counterclockwise starting from $(3, 0)$.

$$\left.\begin{array}{l} x = 3\cos\theta \\ y = 3\sin\theta. \end{array}\right\} \quad 0 \leq \theta \leq 4\pi$$

(b) Once around clockwise starting from $(0, 3)$.

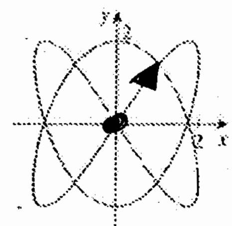
$$\left.\begin{array}{l} x = 3\sin\theta \\ y = 3\cos\theta \end{array}\right\} \quad 0 \leq \theta \leq 2\pi.$$

(OVER)

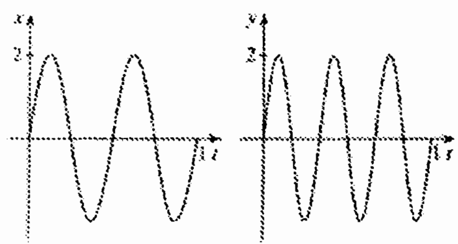
4. Consider the graphs below, and complete all three bullet items: 8 pts

- The equations $x = f(t)$ and $y = g(t)$ graphed in (b) match the parametric curve (circle one): I II III IV.
- ON THE PARAMETRIC GRAPH YOU IDENTIFIED, DRAW A LARGE DOT AT THE POINT IN THE X-Y PLANE CORRESPONDING TO $t = 0$, using the corresponding parametric equations $x = f(t)$ and $y = g(t)$ in (b) as a guide.
- ON THE SAME PARAMETRIC GRAPH, DRAW AN ARROW TO INDICATE THE DIRECTION OF INCREASING T, using the corresponding parametric equations $x = f(t)$ and $y = g(t)$ in (b) as a guide.

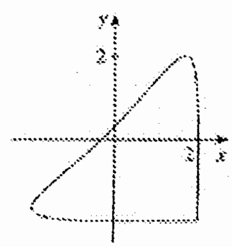
I



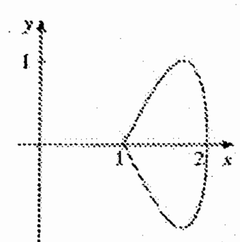
(b)



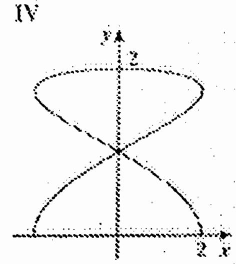
II



III



IV



5. Set up but do not evaluate an integral for the length of the curve $x = \ln t$, $y = \sqrt{1+t}$, $1 \le t \le 5$.

8 pts

$$\int_1^5 \sqrt{(dx/dt)^2 + (dy/dt)^2}\, dt$$

$$= \int_1^5 \sqrt{\left(\frac{1}{t}\right)^2 + \left(\frac{1}{2}\frac{1}{\sqrt{1+t}}\right)^2}\, dt$$

6. Set up but do not evaluate an integral for the area of the surface obtained by rotating the parametric curve $x = a\cos^3(\theta)$, $y = a\sin^3(\theta)$, $0 \le \theta \le \pi/2$, about the x-axis.

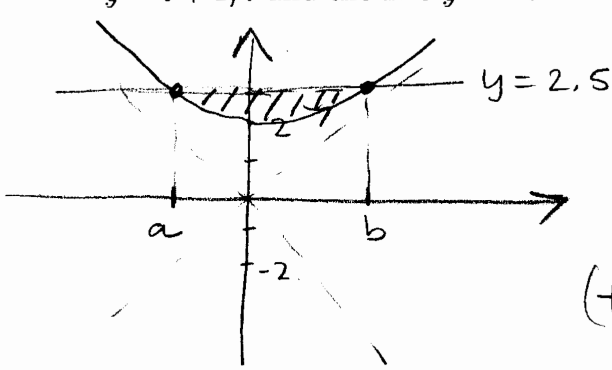
8 pts

$$\int_0^{\pi/2} 2\pi y \sqrt{(dx/d\theta)^2 + (dy/d\theta)^2}\, d\theta$$

$$= \int_0^{\pi/2} 2\pi (a\sin^3\theta) \sqrt{[-3a\cos^2\theta \sin\theta]^2 + [3a\sin^2\theta\cos\theta]^2}\, d\theta$$

7. Set up but do not evaluate an integral for the the area bounded by the curve $x = t - 1/t$, $y = t + 1/t$ and the line $y = 2.5$.

8 pts



$$A = \int_{t_1}^{t_2} (2.5 - y)\frac{dx}{dt}\, dt$$

$$A = \int_{1/2}^{2} \left(2.5 - t - \frac{1}{t}\right)\left(1 + \frac{1}{t^2}\right) dt$$

$$\left(t + \frac{1}{t} = \frac{5}{2}\right) t2$$

$$2t^2 - 5t + 2 = 0$$

$$(2t-1)(t-2) = 0$$

$$t = \frac{1}{2}, 2$$

$$y^2 - x^2 = t^2 + \frac{2}{\cancel{t}}\cancel{t} + \frac{1}{t^2} - \left(t^2 - \frac{2}{\cancel{t}}\cancel{t} + \frac{1}{t^2}\right) = 4$$

8. Set up but do not evaluate an integral for the length of the polar curve $r = 3\sin\theta$, $0 \le \theta \le \pi/3$.

8 pts

$$L = \int_0^{\pi/3} \sqrt{r^2 + (dr/d\theta)^2}\, d\theta$$

$$= \int_0^{\pi/3} \sqrt{9\sin^2\theta + 9\cos^2\theta}\, d\theta$$

(OVER)

9. Find a Cartesian equation for the curve represented by the polar equation $r = \dfrac{1}{\sin\theta + \cos\theta}$.

6 pts

$r(\sin\theta + \cos\theta) = 1$

$r\sin\theta + r\cos\theta = 1$

$y + x = 1$

$\boxed{y = 1 - x}$

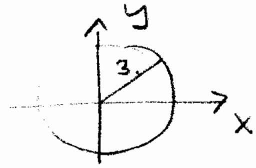
graph is a line with slope $= -1$ & y-int $= 1$.

10. Find a polar equation for the curve represented by the Cartesian equation $x^2 + y^2 = 9$.

6 pts

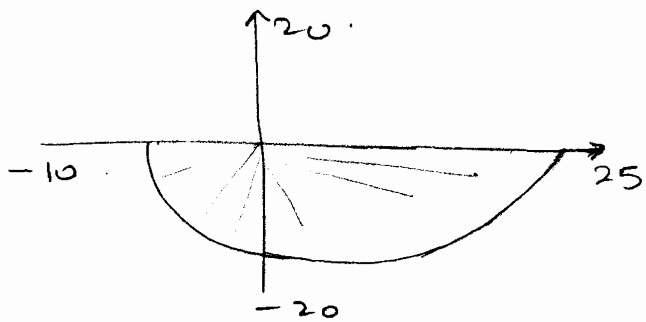
$r^2 = 9$.

$\boxed{r = 3}$



11. Sketch the polar curve $r = e^{\theta/2}$ for $\pi \le \theta \le 2\pi$, and set up but do not evaluate an integral for the area of the region between the curve and the x-axis.

8 pts



$$A = \int_{\pi}^{2\pi} \frac{1}{2}\left[e^{\theta/2}\right]^2 d\theta$$

12. Find the EXACT slope of the tangent line to the polar curve $r = \ln\theta$ at $\theta = e$.

10 pts

$$\left.\frac{dy}{dx}\right|_{\theta=e} = \left.\frac{dy/d\theta}{dx/d\theta}\right|_{\theta=e}$$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}(r\sin\theta) = \frac{d}{d\theta}(\ln\theta\cdot\sin\theta)$$
$$= \ln\theta\cdot\cos\theta + \sin\theta\,\frac{1}{\theta}$$

$$\frac{dx}{d\theta} = \frac{d}{d\theta}(r\cos\theta) = \frac{d}{d\theta}(\ln\theta\cdot\cos\theta)$$
$$= \ln\theta(-\sin\theta) + \cos\theta\cdot\frac{1}{\theta}.$$

$$\Rightarrow \boxed{m = \frac{\cos e + \sin e\cdot\frac{1}{e}}{-\sin e + \cos e\cdot\frac{1}{e}}}$$