Calc II
T2 . Sp 2007
Dr Laura Gross

Test 1

1. Evaluate the integral $\int \frac{e^{2x}}{\sqrt{1-e^{4x}}}\,dx$.

15 pts

$u = e^{2x}$

$du = 2e^{2x}\,dx.$

$$\frac{1}{2}\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2}\sin^{-1}u + c$$

$$= \frac{1}{2}\sin^{-1}(e^{2x}) + c.$$

88.5
HW (1)

2. Evaluate the integral $\int \frac{\sin x + \sec x}{\tan x}\,dx.$

15 pts

$$= \int \frac{\left(\sin x + \frac{1}{\cos x}\right) \times \cos x}{\frac{\sin x}{\cos x} \times \cos x}\,dx$$

$$= \int \frac{\sin x \cos x + 1}{\sin x}\,dx = \int \left(\frac{\cancel{\sin x}\cos x}{\cancel{\sin x}} + \frac{1}{\sin x}\right)dx$$

$$= \int (\cos x + \csc x)\,dx = \int \cos x\,dx + \int \csc x\,dx$$

$$= \sin x + \ln|\csc x - \cot x| + c.$$

check my antiderivative for $\csc x$:

$$\frac{d}{dx}\ln|\csc x - \cot x| = \frac{-\csc x\cot x + \csc^2 x}{\csc x - \cot x} =$$

$$\csc x\,[\csc x - \cot x]/[\csc x - \cot x] = \csc x \checkmark$$

(OVER)

ch 8 review (11)

Dr. Laura Gross

3. Evaluate the integral $\int_1^2 \frac{\sqrt{x^2-1}}{x}\,dx$ EXACTLY.

15 pts

$x = \sec\theta$

$dx = \sec\theta\tan\theta\, d\theta$

$\sqrt{x^2-1} = \tan\theta$

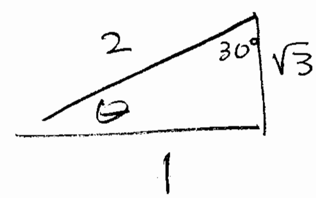
$$\int_0^{\pi/3} \frac{\tan\theta}{\cancel{\sec\theta}}\, \cancel{\sec\theta}\tan\theta\, d\theta = \int_0^{\pi/3} \tan^2\theta\, d\theta =$$

$$= \int_0^{\pi/3} (\sec^2\theta - 1)\, d\theta = (\tan\theta - \theta)\Big|_0^{\pi/3}$$

$x = 1 \Rightarrow \theta = 0$

$x = 2 \Rightarrow \theta = \pi/3$

$$= \sqrt{3} - \frac{\pi}{3}$$



$\sec\theta = 2$

Laura Gross

chap 8 review (25 ?) (first part)

4. Write out the form of a partial-fraction decomposition of the function below. **Determine** the numerical values of the coefficients. DO NOT INT

$$\frac{3x^3 - x^2 + 6x - 4}{(x^2+1)(x^2+2)}$$

15 pts

$$= \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+2} = \frac{(Ax+B)(x^2+2)+(Cx+D)(x^2+1)}{(x^2+1)(x^2+2)}$$

$$\Rightarrow Ax^3 + Bx^2 + 2Ax + 2B +$$
$$+ Cx^3 + Dx^2 + Cx + D = 3x^3 - x^2 + 6x - 4$$

$A + C = 3 \to C = 3 - A$ $\quad \boxed{C = 0}$

$B + D = -1$ $\quad D = -1 - B$. $\quad \boxed{D = 2}$.

$2A + C = 6$ $\quad 2A + 3 - A = 6 \to \boxed{A = 3}$ $\quad 2B - 1 - B = -4$

$2B + D = -4$ $\quad \boxed{B = -3}$

quiz

5. Evaluate the integral $\int \ln(x^2 + 9x + 14)\, dx$.

15 pts

$$\int [\ln(x+7) + \ln(x+2)]\, dx$$

$$\int \ln y\, dy \qquad u = \ln y \quad dv = dy$$
$$du = \frac{1}{y}dy \quad v = y$$

$$= y \ln y - \int dy = y\ln y - y.$$

$$\int \ln(x+7)\,dx \qquad y = x+7 \to dy = dx$$

$$= \int \ln y\, dy = y\ln y - y = (x+7)\ln(x+7) - (x+7)$$

$$\int \ln(x+2)\,dx \qquad y = x+2 \to dy = dx$$

$$= \int \ln y\, dy = y \ln y - y = (x+2)\ln(x+2) - (x+2)$$

$$I = (x+7)\ln(x+7) - (x+7) +$$
$$+ (x+2)\ln(x+2) - (x+2) + C$$

(OVER)

Laura Gross.

§8.8 (2)

6. Circle each integral that is improper. **DO NOT DO ANY INTEGRATION.**

12 pts

(a) $\int_1^2 \frac{1}{2x-1}\,dx$ $x = 1/2$

(b) $\int_0^1 \frac{1}{2x-1}\,dx$ $1/2$

(c) $\int_{-\infty}^{\infty} \frac{\sin x}{x^2+1}\,dx$

(d) $\int_1^2 \ln(x-1)\,dx$

§8.8 HW(5)

7. Determine whether the integral $\int_1^{\infty} \frac{1}{(3x+1)^2}\,dx$ is convergent or divergent. Evaluate it if it is convergent. Write DIVERGENT if it is divergent. Show your work.

13 pts

$$\lim_{t\to\infty} \int_1^t (3x+1)^{-2}\,dx \qquad u = 3x+1 \quad du = 3\,dx$$

$$= \lim_{t\to\infty} \frac{1}{3} \left.\frac{(3x+1)^{-1}}{-1}\right|_1^t$$

$$= \lim_{t\to\infty} -\frac{1}{3}\left[\frac{1}{3t+1} - \frac{1}{4}\right] = \frac{1}{12}$$