We'll be interested in linear equations involving an unknown element of a vector space S. How do we define a vector space?

Definition: A set S is a **vector space over** $\mathbb{C}$ if we

(i) let $x,\ y,\ z \in S$,

(ii) let $\alpha,\ \beta \in \mathbb{C}$,

(iii) define addition of elements of S: $x + y$;
define scalar multiplication: αx, and

(iv) check that the following 10 properties hold:

1. **Closure under addition**: $x + y \in S$
2. **Closure under scalar multiplication**: $\alpha x \in S$
3. **Existence of an additive identity in** S
   (a) There is an element 0 in the set S
   (b) such that $x + 0 = x$
4. **Existence of additive inverses in** S
   (a) There is an element $-x$ in the set S
   (b) such that $x + -x = 0$
5. $x + y = y + x$ (Addition is commutative.)
6. $(x + y) + z = x + (y + z)$ (Addition is associative.)
7. $(\alpha + \beta)x = \alpha x + \beta x$ (An element of S can be distributed over a sum of scalars.)
8. $(\alpha\beta)x = \alpha(\beta x)$ (Scalar multiplication is associative.)
9. $\alpha(x+y) = \alpha x+\alpha y$ (A scalar can be distributed over a sum of elements of S.)
10. $1x = x$