

In our unit on **linear analysis**, we will look at the unifying features of linear algebra and linear differential equations. Linear systems are easier to analyze than nonlinear ones because, as Strogatz<sup>1</sup> puts it, “*linear systems can be broken down into parts*. Then each part can be solved separately and finally recombined to get the answer. This idea allows a fantastic simplification of complex problems, and underlies such methods as normal modes, Laplace transforms, superposition arguments, and Fourier analysis. In this sense, a linear system is precisely equal to the sum of its parts.

“But many things in nature don’t act this way. Whenever parts of a system interfere, or cooperate, or compete, there are nonlinear interactions going on. Most of everyday life is nonlinear, and the principle of superposition fails spectacularly. If you listen to your two favorite songs at the same time, you won’t get double the pleasure! Within the realm of physics, nonlinearity is vital to the operation of a laser, the formation of turbulence in a fluid, and the superconductivity of Josephson junctions.”

So if linear systems are easier to analyze, let’s analyze them! Luckily for engineers, scientists, and mathematicians, many physical systems are linear. Equally or even more importantly, techniques from linear analysis can be applied in carefully controlled ways to get insights into nonlinear systems, as well.

We’ll discuss the concepts of vector spaces, linear operators, linear equations, basis and dimension, orthogonality, eigenvalues and eigenvectors, adjoint operators, and solvability. Linear analysis will lead us into our next major unit: systems of differential equations.

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<sup>1</sup>*Nonlinear Dynamics and Chaos*, Steven H. Strogatz, Westview Press, Cambridge, MA (1994)