

# 3450:438/538:001    **Homework 6**    Fall 2007

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**Course:** Advanced Engineering Math I

**Instructor:** Dr. Laura Gross

**Due date:** Wednesday, October 3, 2007

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1. Prove that the superposition principle holds for all linear homogeneous equations. That is, if  $u$  and  $v$  satisfy the linear homogeneous equation  $L(x) = 0$  then  $\alpha u + \beta v$  also satisfies the equation for any scalars  $\alpha$  and  $\beta$ .
2. Referring to your proof in Problem 1, briefly explain why the superposition principle does not hold for nonlinear equations.
3. Referring to your proof in Problem 1, briefly explain why the superposition principle does not hold for nonhomogeneous equations.
4. Assuming the linear operator  $L$  maps the vector space  $S_1$  into the vector space  $S_2$ , prove that the set  $S$  of solutions to  $L(x) = 0$  is a subspace of  $S_1$ . (This statement is another way to express the superposition principle.)
5. Consider the vector space  $S = \text{Span}(2x + 3, x - 1, -x - 4)$ .
  - (a) Identify two elements of  $S$ .
  - (b) Using the definition of *span*, prove or disprove that  $f(x) = -x + 6 \in S$ .
  - (c) Prove  $S$  is a subspace of the vector space  $P_2$  of polynomials of degree less than or equal to two.<sup>1</sup>
  - (d) Find a basis for  $S$ . Prove it is a basis.
  - (e) What is the dimension of  $S$ ?
6. Let  $S$  be the subspace of the vector space  $\mathcal{R}^3$ , where  $S$  consists of all elements of the form  $(3a, b, b - a)$ .
  - (a) Determine a basis for  $S$ .
  - (b) What is the dimension of  $S$ ?

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<sup>1</sup>Alternatively, you could prove that if  $u$  and  $v$  are elements of an arbitrary vector space  $V$ , then  $\text{Span}(u, v)$  is a subspace of  $V$ .

7. Let  $U$  be the vector space over  $\mathcal{R}$  of  $2 \times 2$  upper triangular matrices with real components, i.e. matrices of the form

$$\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}.$$

- (a) Determine a basis for  $D$ .
- (b) What is the dimension of  $D$ ?
8. Consider the linear homogeneous differential equation

$$y''(x) + y'(x) - 6y(x) = 0.$$

- (a) Verify by substitution that  $y_1(x) = e^{2x}$  and  $y_2(x) = e^{-3x}$  are solutions. Prove the two solutions are linearly independent.
- (b) Using the definition of *span*, explain what it means for  $y_1$  and  $y_2$  to span the vector space of solutions. (They do, in fact, span it.)
- (c) Explain why the following statement, although true, is not a correct answer to (8b): Every linear combination of  $y_1$  and  $y_2$  is also a solution.