3450:438/538:001 **Homework 5** Fall 2007

Course: Advanced Engineering Math I
Instructor: Dr. Laura Gross
Recommended due date: Wednesday, September 26, 2007

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1. Prove or disprove that S is a subspace of $\mathbb{R}^n$ if S is the set of all solutions to the equation $A\mathbf{x} = \mathbf{1}$ (where A is an $m \times n$ real matrix, $\mathbf{x}$ is a column vector in $\mathbb{R}^n$, and $\mathbf{1}$ is a column vector in $\mathbb{R}^m$ whose components are all 1). **Do not solve any linear systems.**

2. Consider the equation for forced vibrations

$$L(u) = F_0 \cos(\omega t), \quad t > 0,$$

where

$$L(u) = m\frac{d^2u}{dt^2} + c\frac{du}{dt} + ku.$$

It governs the evolution in time of the displacement u of a mass m suspended from a spring if the damping constant is c and k is the spring constant. The mass is being forced with amplitude F_0 and frequency ω. The equation has many solutions, depending on the initial conditions.

   (a) Let S be the set of solutions to the above equation $L(u) = F_0 \cos(\omega t)$ that have two continuous derivatives. Prove or disprove that S is a subspace of the vector space $\mathcal{C}^2$ of twice continuously differentiable functions. **Do not solve the differential equation.**

   (b) Prove or disprove that L is a linear operator.

3. Let S be the set of all harmonic functions. Prove or disprove that S is a subspace of the vector space $\mathcal{C}^2$ of functions with all of their second partial derivatives continuous. Recall that a harmonic function $u(x, y)$ has all its second partial differentives continuous and satisfies the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

by definition. **Do not solve any differential equations.**

4. By definition, the gradient of $f(x, y)$ is $\nabla f = <\partial f/\partial x, \partial f/\partial y>$.

   The gradient operator maps the vector space of continuous functions $f(x, y)$ with continuous derivatives with respect to x and y into the vector space of continuous vector functions of the form $<g(x, y), h(x, y)>$.

   Prove or disprove that the gradient is a linear operator.