3450:438/538:001 **Homework 3** Fall 2007

Course: Advanced Engineering Math I
Instructor: Dr. Laura Gross
Recommended due date: Wednesday, September 12, 2007

THIS HOMEWORK IS NOT FOR COURSE CREDIT. However, you need to do problems to learn the material. Also, about 1/3 of your exam will consist of recommended homework problems.

Each answer must use **exact values**. (Do not give only decimal approximations to answers.) Show your work. Recall you can check many of your answers using the complex-variables capabilities of your graphing calculator.

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1. Write the function $w = f(z) = \dfrac{z+i}{z^2+1}$ in the form $w = u(x,y) + iv(x,y)$. **Hint:** Substitute $z = x + iy$ into the function f.

2. Consider the function $w(x,y) = \dfrac{x-1+2iy}{(x-1)^2+y^2}$.

   (a) Note $w(x,y)$ has the form $u(x,y)+iv(x,y)$. Identify $u(x,y)$ and $v(x,y)$.

   (b) Let $z = x + iy$. Show that $x = \dfrac{z+\bar{z}}{2}$, and $y = \dfrac{z-\bar{z}}{2i}$.

   (c) Make the substitution indicated in (2b) to write the $w(x,y)$ as a function of z and $\bar{z}$ instead of x and y.

3. For two complex numbers z_1 and z_2, prove that if $z_1 = z_2 + k2\pi i$, $k \in \mathcal{Z}$, then $e^{z_1} = e^{z_2}$.

4. Show that the complex exponential function $f(z) = e^z$ has period $p = 2\pi i$, i.e. show $f(z+p) = f(z)$.

5. Find a complex number z such that $|\sin(z)| > 1$. **Hint: Look for a purely imaginary z that does the trick.** Note: $|\sin(z)|$ and $|\cos(z)|$ can be made arbitrarily large!

6. Use the definition of the complex sine and cosine functions to prove the sum identity for cosine. Namely, show $\cos(z_1 + z_2) = \cos(z_1)\cos(z_2) - \sin(z_1)\sin(z_2)$.

7. Prove $\overline{(e^z)} = e^{\bar{z}}$.

8. Write $\sin(2i)$ in the form $a + bi$.

9. Find the principal value of $\log(i\sqrt{3} - 1)$. Briefly explain why the complex logarithm is multi-valued.

10. Calculate $\lim_{\omega \to \infty} \rho e^{-i\omega}$ for $\omega \in \mathcal{R}$, or state that the limit does not exist. Show your work using Euler's formula.

11. Identify and classify the isolated singularities, if any, of the following functions:

    (a) $f(z) = \dfrac{1 - \cos(z)}{z}$

    (b) $f(z) = \dfrac{e^z}{(z - (1 + i))^2 (z + 2i)^3}$

    (c) $f(z) = \sin\left(\dfrac{1}{z - 2 + 3i}\right)$