

3450:438/538:001 Homework 2 Fall 2007

Course: Advanced Engineering Math I

Instructor: Dr. Laura Gross

Due date: Wednesday, September 5, 2007

Each answer must use **exact values**. (Do not give only decimal approximations to answers.) Show your work. Recall you can check some of your answers using the complex-variables capabilities of your graphing calculator.

1. For each of the complex numbers below, (i) find $|z|$: the modulus of z , (ii) find a value of $\arg(z)$ in the interval $[0, 2\pi)$, (iii) find $\text{Arg}(z)$ as defined in class, (iv) write z in a *two different polar forms*, (v) write z in a polar form such that all infinitely many of the values of $\arg(z)$ are included, (vi) graph z in the complex plane.

(a) 5

(b) $-2i$

(c) $(1 + i)(-\sqrt{3} + i)$

2. Write $3e^{i3\pi/4}$ in the form $x + iy$. Graph it in the complex plane.

3. For each of the complex numbers below, (i) find a polar form for z , (ii) write z in the form $x + iy$ (iii) graph z in the complex plane.

(a) $(i + 1)^{2i}$

(b) $(\sqrt{3})^{3i-1}$

(c) $(-\sqrt{3} + i)^{20}$

4. Prove $(\bar{z})^k = \overline{z^k}$ for all complex numbers z and all real numbers k . (This fact helps prove that if z is a root of a polynomial with real coefficients, then \bar{z} is also a root.)

5. Let $z = -2 - 3i$. Graph z , $-z$, \bar{z} , and $1/z$ in the complex plane, and label them. Briefly explain the geometric relationship between the graph of each one and the graph of z .

6. For each problem, sketch the set of points in the complex plane that satisfies: (a) $[\Re(z)]^2 < 3$, (b) $|z - i - 1| > 1/2$, (c) $|\Im(z)| \geq 4$, (d) $\pi/4 < \arg(z) \leq 7\pi/4$ (e) $|-1 + 2i - 3z| = 6$, (f) $|z - 1| + |z + 1| = 7$. In Part (f), explain why the graph is an ellipse, using the basic geometric definition of an ellipse and interpreting the moduli as distances.

7. Find all distinct fourth roots of $-1 + i\sqrt{3}$, and graph them.
8. Give a brief geometrical interpretation of complex division, referring to an accompanying sketch. Write your explanation in a couple of complete sentences. (Hint: If $z_1/z_2 = z_3$, what is the relationship among the moduli of z_1 , z_2 and z_3 ? What is the relationship among the angles with the positive real axis?)