3450:438/538:001 **Homework 13** Fall 2007

Course: Advanced Engineering Math I
Instructor: Dr. Laura Gross
Recommended due date: Wednesday, November 21, 2007

THIS HOMEWORK IS NOT FOR COURSE CREDIT. However, you need to do problems to learn the material. Also, about 1/3 of your exam will consist of recommended homework problems.

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1. Let $f = \frac{1}{4}(x+y)^2$.

   (a) Find the gradient vector field ∇f, and sketch it by hand. (Include in the domain all the pairs (x, y) of integers on a grid $[-2, 2] \times [-2, 2]$.)

   (b) Make some observations about the behavior of the vector field.

2. Consider the velocity field $\mathbf{F}(x, y) = \mathbf{i} + x\mathbf{j}$.

   (a) Sketch the vector field by hand, including in the domain all the pairs (x, y) of integers on a grid $[-2, 2] \times [-2, 2]$.

   (b) Lightly sketch some streamlines (flow lines) on top of your graph in Problem (2a). What shape do the flow lines appear to have?

   (c) If parametric equations of the flow lines are $x = x(t)$, $y = y(t)$, what differential equations do these function satisfy? Deduce that $dy/dx = x$.

   (d) If a particle starts at the origin in the velocity field given by $\mathbf{F}$, find an equation of the path it follows. Revise the streamlines in your figure from Problem (2a) if necessary to show this path accurately.

3. Find the work done by the force field $\mathbf{F}(x, y) = x\sin(y)\mathbf{i} + y\mathbf{j}$ on a particle that moves along the parabola $y = x^2$ from $(-1, 1)$ to $(2, 4)$.

4. Consider that the magnitude of the force exerted by an electric charge (at the origin) on a charged particle (at the point (x, y, z)) is inversely proportional to the distance between the charges squared. Show that the force field has the form $\mathbf{F}(\mathbf{r}) = K\mathbf{r}/||\mathbf{r}||^3$, where $\mathbf{r}$ is the position vector $\langle x, y, z\rangle$, and K is a constant. Find the work done by the electric field on a particle that moves along a straight line from $(2, 0, 0)$ to $(2, 1, 5)$.