3450:438/538:001 **Homework 11** Fall 2007

Course: Advanced Engineering Math I
Instructor: Dr. Laura Gross
Recommended due date: Wednesday, November 7, 2007

THIS HOMEWORK IS NOT FOR COURSE CREDIT. However, you need to do problems to learn the material. Also, about 1/3 of your exam will consist of recommended homework problems.

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1. Consider the system $x_1' = 2x_1 - 5x_2$, $x_2' = x_1 - 2x_2$.

   (a) Classify the critical point $(0, 0)$ as to type and stability.

   (b) Sketch a set of approximate trajectories in the phase plane, representative of the behavior of solutions to the linear system of differential equations.

   (c) Without solving, but using your answer to (1b), draw an approximate sketch the solution to the differential equation subject to the initial condition $x_1(0) = 5$, $x_2(0) = 4$.

   (d) Find the real-valued general solution $x_1(t)$, $x_2(t)$.

   (e) Solve the initial-value problem with the initial condition in (1c).

   (f) Verify the solution in (1e) by substituting it back into the system and the initial condition.

   (g) For the solution in (1e), plot x_1 versus t and x_2 versus t on the same set of axes, and label them. Compare the graph to (1c).

   (h) Plot x_2 vs x_1 parametrically. Remember to include an arrow to indicate the direction of increasing t. Compare the graph to (1c).

2. Consider the system $x_1' = -\frac{1}{2}x_1 + x_2$, $x_2' = -x_1 - \frac{1}{2}x_2$.

   (a) Find the general real-valued solution $x_1(t)$, $x_2(t)$. (**Recommended:** Verify the solution by substituting it back into the system.)

   (b) Draw a sketch of a representative set of trajectories in the phase plane.

   (c) Classify the critical point $(0, 0)$ as to type and stability.

3. Do "attached" Problem 33.

(OVER)

4. Consider the system $x_1' = 2x_1$, $x_2' = 2x_2$.

   (a) Find the general real-valued solution $x_1(t)$, $x_2(t)$. Be sure to use two linearly independent eigenvectors.

   (b) Draw a sketch of a representative set of trajectories in the phase plane.

   (c) Classify the critical point $(0, 0)$ as to type and stability per our class handout "Solutions of systems of differential equations and classification of the origin."

5. Consider the system $x_1' = x_1 - x_2$, $x_2' = x_1 + 3x_2$.

   (a) Show that it is not possible to find two linearly independent eigenvectors to construct the general solution, contrasting this system with the one in the previous problem.

   (b) Classify the critical point $(0, 0)$ as to type and stability per our class handout "Solutions of systems of differential equations and classification of the origin."

6. Consider the system $x_1' = 2x_1 + x_2$, $x_2' = 2x_1 + x_2$.

   (a) Find all the equilibrium solutions, and graph them in the phase plane.

   (b) Find the general real-valued solution $x_1(t)$, $x_2(t)$.

   (c) Draw a sketch of a representative set of trajectories in the phase plane.

   (d) Classify the equilibria as to stability.

   (e) Briefly explain why this system does not fall info any of the cases on our class handout "Solutions of systems of differential equations and classification of the origin."