# CLASSIFICATION OF CRITICAL POINTS FOR ALMOST LINEAR SYSTEMS

3450:438/538-001 Advanced Engineering Math I
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Consider the almost linear system $x'(t) = F(x,y)$, $y'(t) = G(x,y)$ with isolated critical point $(x_0, y_0)$. Let $\mathbf{u}' = L\mathbf{u}$ be the system linearized about the critical point.

Let $\lambda_1$ and $\lambda_2$ be the eigenvalues of $L$. Then the type and stability of the critical point $\mathbf{u} = (0,0)$ of the linear system ("lin") and the type and stability of the critical point $\mathbf{x} = (x_0, y_0)$ of the almost linear system ("nonlin") are as shown in the table below[1].

| $\lambda_1$, $\lambda_2$ | Type (lin) | Stability (lin) | Type (nonlin) | Stability (nonlin) |
|---|---|---|---|---|
| $\lambda_1 > \lambda_2 > 0$ | IN | Unstable | IN | Unstable |
| $\lambda_1 < \lambda_2 < 0$ | IN | Asy. stable | IN | Asy. stable |
| $\lambda_2 < 0 < \lambda_1$ | Saddle | Unstable | Saddle | Unstable |
| $\lambda = \alpha \pm i\beta,\ \alpha > 0$ | SpP | Unstable | SpP | Unstable |
| $\lambda = \alpha \pm i\beta,\ \alpha < 0$ | SpP | Asy. stable | SpP | Asy. stable |
| $\lambda = \alpha \pm i\beta,\ \alpha = 0$ | C | Stable | C or SpP | Indeterminate |
| $\lambda_1 = \lambda_2 > 0$ | PN or IN | Unstable | IN or PN or SpP | Unstable |
| $\lambda_1 = \lambda_2 < 0$ | PN or IN | Asy. stable | IN or PN or SpP | Asy. stable |

[1]NSpP = spiral point; C = center; IN = improper node; PN = proper node