Hw 5 - A Soln

A&M I
Fall 2007
Gross.

(1) Disprove S is a subspace of $\mathbb{R}^n$.

(1) $S \subseteq \mathbb{R}^n$ ✓

(2). let $\underline{x}, \underline{y} \in S$:

$$A\underline{x} = \underline{1} \ , \ A\underline{y} = \underline{1} \qquad (*)$$

let $\alpha \in \mathbb{R}$.

$$\underline{x}+\underline{y} \overset{?}{\in} S \iff A(\underline{x}+\underline{y}) \overset{?}{=} \underline{1}$$

$$\iff A\underline{x} + A\underline{y} \overset{?}{=} \underline{1} \underset{\text{by } (*)}{\iff} \underline{1} + \underline{1} \overset{?}{=} \underline{1}$$

No: If you add two vectors whose components are all 1, you don't get a vector whose components are all 1. (You get a vector whose components are all 2.)

So S is not closed under addition.

So S is not a subspace.

(2a). Disprove S is a subspace.

$S \subset \mathcal{C}^2$, but S is not closed under scalar mult. (& not closed under addition).

Let $v \in S$.

$$L(v) = mv''(t) + cv'(t) + kv = F_0 \cos\omega t.$$

Let $\alpha \in \mathbb{R}$.

$$\alpha v \overset{?}{\in} S \iff L(\alpha v) \overset{?}{=} F_0 \cos\omega t$$

$$\begin{aligned} L(\alpha v) &= m(\alpha v(t))'' + c(\alpha v(t))' + k(\alpha v(t)) \\ &= \alpha [mv''(t) + cv'(t) + kv(t)] \\ &= \alpha F_0 \cos\omega t \\ &\neq F_0 \cos\omega t \text{ in general.} \end{aligned}$$

(2b). L is a linear operator:

$$\begin{aligned} L(\alpha u(t) + \beta v(t)) &= m(\alpha u(t) + \beta v(t))'' + c(\alpha u(t) + \beta v(t))' + k(\alpha u(t) + \beta v(t)) \\ &= m(\alpha u'' + \beta v'') + c(\alpha u' + \beta v') + k(\alpha u + \beta v). \\ &= \alpha[mu'' + cu' + ku] + \beta[mv'' + cv' + kv] \\ &= \alpha L(u(t)) + \beta L(v(t)) \quad \checkmark \end{aligned}$$

(3). Prove S is a subspace.

① $S \subseteq C^2$. ✓

② Check closure.

(i) Let $f, g \in S$.

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0, \quad \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} = 0.$$

(ii) Let $\alpha \in \mathbb{R}$.

(a) $f+g \overset{?}{\in} S \iff \frac{\partial^2}{\partial x^2}(f+g) + \frac{\partial^2}{\partial y^2}(f+g) \overset{?}{=} 0.$

$$\frac{\partial^2}{\partial x^2}(f+g) + \frac{\partial^2}{\partial y^2}(f+g) =$$

$$= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} = 0 + 0 = 0 \checkmark$$

S is closed under addition.

(b) $\alpha f \overset{?}{\in} S \iff \frac{\partial^2}{\partial x^2}(\alpha f) + \frac{\partial^2}{\partial y^2}(\alpha f) \overset{?}{=} 0$

$$\frac{\partial^2}{\partial x^2}(\alpha f) + \frac{\partial^2}{\partial y^2}(\alpha f) = \alpha \frac{\partial^2 f}{\partial x^2} + \alpha \frac{\partial^2 f}{\partial y^2} =$$

$$= \alpha \left[\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \right] = \alpha 0 = 0 \checkmark$$

S is closed under scalar mult.

So S is a subspace.

(4) $\underline{\nabla}\left(\alpha f(x,y) + \beta g(x,y)\right) \stackrel{?}{=} \alpha \underline{\nabla} f(x,y) + \beta \underline{\nabla} g(x,y)$.

$$\underline{\nabla}\left(\alpha f(x,y) + \beta g(x,y)\right)$$

$$= \left\langle \frac{\partial}{\partial x}\left(\alpha f(x,y) + \beta g(x,y)\right), \frac{\partial}{\partial y}\left(\alpha f(x,y) + \beta g(x,y)\right) \right\rangle$$

$$= \left\langle \frac{\partial}{\partial x}\left(\alpha f(x,y)\right) + \frac{\partial}{\partial x}\left(\beta g(x,y)\right), \frac{\partial}{\partial y}\left(\alpha f(x,y)\right) + \frac{\partial}{\partial y}\left(\beta g(x,y)\right) \right\rangle$$

$$= \left\langle \alpha \frac{\partial f}{\partial x} + \beta \frac{\partial g}{\partial x}, \alpha \frac{\partial f}{\partial y} + \beta \frac{\partial g}{\partial y} \right\rangle.$$

$$= \left\langle \alpha \frac{\partial f}{\partial x}, \alpha \frac{\partial f}{\partial y} \right\rangle + \left\langle \beta \frac{\partial g}{\partial x}, \beta \frac{\partial g}{\partial y} \right\rangle.$$

$$= \alpha \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle + \beta \left\langle \frac{\partial g}{\partial x} + \frac{\partial g}{\partial y} \right\rangle.$$

$$= \alpha \underline{\nabla} f(x,y) + \beta \underline{\nabla} g(x,y) \quad \checkmark$$

the gradient is a linear operator.