HW 4-A Sol'n

AEM I
F2007
Gross.

(1) $\frac{dw}{dz} = \cos(e^{2z})\, e^{2z}\, 2.$

(2) $w(z) = w(x+iy) = x - iy \Rightarrow \begin{cases} u(x,y) = x \\ v(x,y) = -y \end{cases}$

$\frac{\partial u}{\partial x} = 1 \neq -1 = \frac{\partial v}{\partial y}$ C-R eq$\underline{\text{ns}}$ do not hold $\Rightarrow$ function is not differentiable.

(3) $u(x,y) = x^3 - 3xy^2, \quad 3x^2y - y^3 = v(x,y).$

$$\left.\begin{aligned} \frac{\partial u}{\partial x} &= 3x^2 - 3y^2 = \frac{\partial v}{\partial y} \checkmark \\ \frac{\partial u}{\partial y} &= -6xy = -\frac{\partial v}{\partial x}. \checkmark \end{aligned}\right\}\text{C-R eq'ns hold.}$$

$u, v, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \& \frac{\partial v}{\partial y}$ are all continuous. Therefore f is differentiable.

(b). $\left(\frac{z+\bar{z}}{2}\right)^3 - 3\left(\frac{z+\bar{z}}{2}\right)\left(\frac{z-\bar{z}}{2i}\right)^2 +$

$+ i\left\{3\left(\frac{z+\bar{z}}{2}\right)^2\left(\frac{z-\bar{z}}{2i}\right) - \left(\frac{z-\bar{z}}{2i}\right)^3\right\} = \left(\text{with some scratchwork}\right)$

$= \frac{1}{8}\left[z^3 + \cancel{3z^2\bar{z}} + \cancel{3\bar{z}^2 z} + \cancel{\bar{z}^3}\right] + \frac{3}{8}\left(z^3 - \cancel{z^2\bar{z}} - \cancel{z\bar{z}^2} + \cancel{\bar{z}^3}\right)$

$+ \frac{3}{8}\left(z^3 + \cancel{z^2\bar{z}} - \cancel{z\bar{z}^2} - \cancel{\bar{z}^3}\right)$

$+ \frac{1}{8}\left[z^3 - \cancel{3z^2\bar{z}} + \cancel{3\bar{z}^2 z} - \cancel{\bar{z}^3}\right] = z^3 = f(z)$

Note if you do (b) first it's easy to see that $f(z)$ is differentiable: $f'(z) = 3z^2$.

(4) (i) Let $A, B \in S$.

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{bmatrix}, \quad B = \begin{bmatrix} b_1 & b_2 & b_3 \\ b_4 & b_5 & b_6 \end{bmatrix}, \quad \begin{array}{l} a_i \geq 0 \\ b_i \geq 0 \\ i = 1, 2, \dots, 6 \end{array}$$

(ii) Let $\alpha \in \mathbb{R}$.

(iii) Define addition & scalar mult as usual for matrices.

(iv) Check the properties.

(2) $\alpha A = \begin{bmatrix} \alpha a_1 & \alpha a_2 & \alpha a_3 \\ \alpha a_4 & \alpha a_5 & \alpha a_6 \end{bmatrix}$.

The entries are non necessarily non-negative.

S is not closed under scalar mult.

So S is not a v.s.

(5) (1) $S \subseteq \mathbb{C}^4$ ✓ (2) check closure:

(i) Let $\underline{u}, \underline{v} \in S$.

$$\left.\begin{array}{l} \underline{u} = (x_1, y_1, 0, x_1 + y_1) \\ \underline{v} = (x_2, y_2, 0, x_2 + y_2) \end{array}\right\} x_i, y_i \in \mathbb{C}.$$

(ii) Let $\alpha \in \mathbb{C}$

(a). $\underline{u} + \underline{v} \overset{?}{\in} S$

$\underline{u} + \underline{v} = (x_1 + x_2, y_1 + y_2, 0, x_1 + x_2 + y_1 + y_2) \in S$

S is closed under addition.

(b) $\alpha \underline{u} \overset{?}{\in} S$

$\alpha \underline{u} = (\alpha x_1, \alpha y_1, 0, \alpha x_1 + \alpha y_1) \in S$

S is closed under scalar mult.

So S is a subspace of $\mathbb{C}^4$.

(6) ① $S \subseteq \mathcal{C}[a,b]$ ✓ ② Check closure:

(i) Let $g, h \in S$

$$g(b) = g(a) + 3$$
$$h(b) = h(a) + 3$$

(ii) Let $\alpha \in \mathbb{R}$.

(a). $g + h \overset{?}{\in} S$.

$$\iff (g+h)(b) \overset{?}{=} (g+h)(a) + 3.$$

$(g+h)(b) = g(b) + h(b)$ ← by def of addition of functions

$$= g(a) + 3 + h(a) + 3$$
$$= (g+h)(a) + 6$$
$$\neq (g+h)(a) + 3.$$

S is not closed under addition, So S is not a subspace of $\mathcal{C}[a,b]$.

Another way: If S is a subspace then S is itself a vector space. However S does not contain the zero function $\Phi(t) \equiv 0$ (since $\Phi(b) = 0 \neq 0 + 3 = \Phi(a) + 3$). Therefore S is not a v.s., so S is not a subspace.