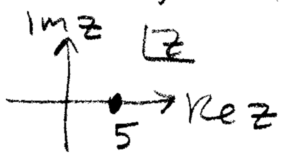
HW2 – A sol^n AEM I Gross

(1). (a) $z = 5$

(i) $|z| = 5$, (ii) $\arg(z) = 0$, (iii) $\operatorname{Arg} z = 0$,

(iv) $z = 5,\ 5e^{2\pi i}$, (v) $z = 5e^{k2\pi i}$, $k \in \mathbb{Z}$,

(vi)



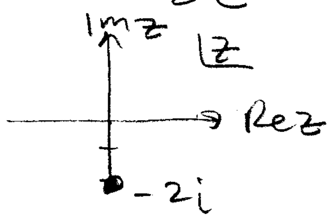
(b) $z = -2i$

(i) $|z| = 2$, (ii) $\arg z = \frac{3\pi}{2}$, (iii) $\operatorname{Arg} z = \frac{-\pi}{2}$,

(iv) $z = 2e^{3\pi i/2},\ 2e^{-\pi i/2}$

(v) $z = 2e^{-\pi i/2 + k2\pi i}$, $k \in \mathbb{Z}$

(vi)



(c) $(1+i)(-\sqrt{3}+i) = (-\sqrt{3}-1) + i(1-\sqrt{3}) = z$

(i) $|z| = \sqrt{(1+\sqrt{3})^2 + (1-\sqrt{3})^2} = \sqrt{8}$.

(ii) $\arg z = \tan^{-1}\left(\frac{-1+\sqrt{3}}{+\sqrt{3}+1}\right) + \pi$

(iii) $\operatorname{Arg} z = \tan^{-1}\left(\frac{-1+\sqrt{3}}{\sqrt{3}+1}\right) - \pi$

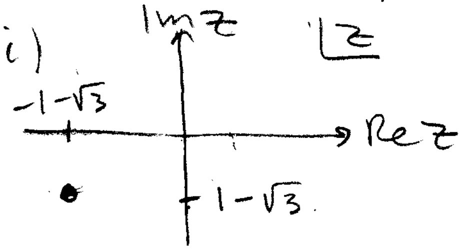
(iv) $z = \sqrt{8}\exp\left(i\left(\tan^{-1}\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) + \pi\right)\right)$,

$\sqrt{8}\exp\left(i\left(\tan^{-1}\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) + 3\pi\right)\right)$

(v) $z = \sqrt{8}\exp\left(i\left(\tan^{-1}\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) + (2k+1)\pi\right)\right)$,

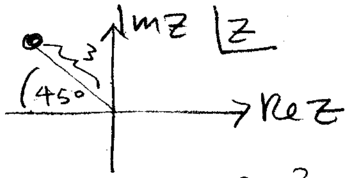
$k \in \mathbb{Z}$.

(vi)



HW 2, 2

(2). $z = 3e^{i3\pi/4}$



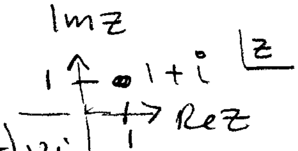
$= x + iy$

$2x^2 = 9$

$x = \frac{3}{\sqrt{2}}$

$= \frac{-3}{\sqrt{2}} + i\frac{3}{\sqrt{2}}$

(3)(a) $z = (i+1)^{2i}$



$= \left(\sqrt{2}\, e^{i(\pi/4 + k2\pi)}\right)^{2i} \quad k \in \mathbb{Z}$

Take $k=0$ to get principal value:

$\sqrt{2}^{\,2i}\, e^{-\pi/2}$

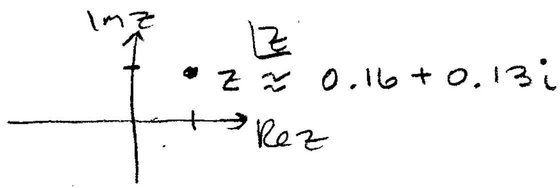
$= \left(e^{\ln\sqrt{2} + ik2\pi}\right)^{2i} e^{-\pi/2}, \quad k \in \mathbb{Z}$

Take $k=0$ to get principal value:

(i) $e^{i2\ln\sqrt{2}}\, e^{-\pi/2} = e^{i2\frac{1}{2}\ln 2}\, e^{-\pi/2}$

(ii) $e^{-\pi/2}\cos(\ln 2) + i e^{-\pi/2}\sin(\ln 2)$

(iii)



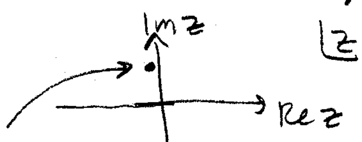
(b) $z = \sqrt{3}^{\,3i-1} = \sqrt{3}^{\,3i}\,\sqrt{3}^{\,-1} = \frac{1}{\sqrt{3}}\sqrt{3}^{\,3i}$

$= \frac{1}{\sqrt{3}}\left(e^{\ln\sqrt{3} + ik2\pi}\right)^{3i}$ Take $k=0$ for principal value

(i) $\frac{1}{\sqrt{3}}\, e^{i3\ln\sqrt{3}}$

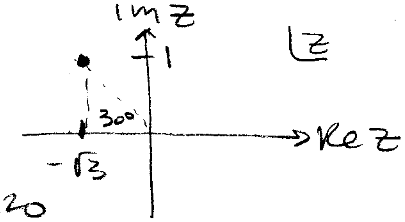
(ii) $\frac{1}{\sqrt{3}}\cos(3\ln\sqrt{3}) + i\frac{1}{\sqrt{3}}\sin(3\ln\sqrt{3})$

(iii)



$z \approx -0.045 + i0.58$

(3c) $(-\sqrt{3}+i)^{20}$



$$= \left(2e^{5\pi/6\, i + ik2\pi}\right)^{20}$$

$$= 2^{20} e^{100\pi/6\, i} \quad \text{(i)}.$$

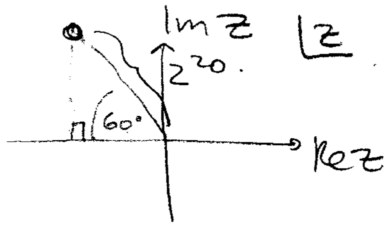
(ii) $2^{20}\cos\frac{\cancel{100}^{50}\pi}{\cancel{6}_{3}} + i2^{20}\sin\frac{\cancel{100}^{50}\pi}{\cancel{6}_{3}}$

$$= 2^{20}\left(-\frac{1}{2}\right) + i\,2^{20}\left(\frac{\sqrt{3}}{2}\right)$$

$$= -2^{19} + i\,2^{19}\sqrt{3}$$

$$\begin{array}{r} 16 \\ 3\overline{)50} \\ \underline{3} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

$\frac{50\pi}{6} = 16\pi + \frac{2}{3}\pi$



(4). $z = re^{i\phi}$

$(\bar{z})^k = (re^{-i\phi})^k = r^k e^{-i\phi k}$

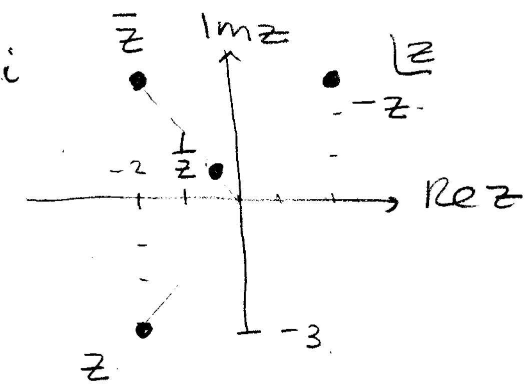
$\overline{(z^k)} = \overline{(re^{i\phi})^k} = \overline{r^k e^{i\phi k}} = r^k e^{-i\phi k}$. $=$ ✓

(5). $z = -2-3i$

$= \sqrt{13}\, e^{i\phi}$

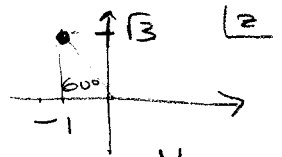
$\Downarrow$

$\frac{1}{z} = \frac{1}{\sqrt{13}} e^{-i\phi}$



Get $\bar{z}$ by reflecting z across the real axis. Get $-z$ by reflecting across real & imaginary axes. Get $\frac{1}{z}$ by reflecting across the real axis & inverting the modulus.

(7). $z = -1 + i\sqrt{3}$



$$z^{1/4} = \left(2e^{i(2\pi/3 + k2\pi)}\right)^{1/4}$$

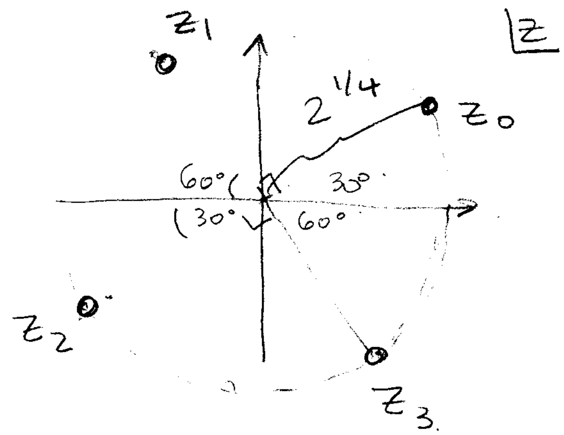
$$= 2^{1/4} e^{i\frac{2\pi + 6\pi k}{12}} \qquad k = 0, 1, 2, 3$$

$$z_0 = 2^{1/4} e^{2\pi/12\, i}$$

$$z_1 = 2^{1/4} e^{\cancel{8}^{2}\pi i/\cancel{12}_{3}}$$

$$z_2 = 2^{1/4} e^{14\pi i/12}$$

$$z_3 = 2^{1/4} e^{20\pi i/12}$$



(8). $z_1 = \rho_1 e^{i\phi_1}$

$z_2 = \rho_2 e^{i\phi_2}$

$$z_3 = \frac{z_1}{z_2} = \frac{\rho_1 e^{i\phi_1}}{\rho_2 e^{i\phi_2}} = \frac{\rho_1}{\rho_2} e^{i(\phi_1 - \phi_2)}.$$

To graph $z_3 = \frac{z_1}{z_2}$, note $|z_3|$ is the quotient $\frac{|z_1|}{|z_2|}$ of the moduli of z_1 & z_2, & $\arg(z_3)$ is the difference $\arg z_1 - \arg z_2$ of the arguments of z_1 & z_2, e.g.