HW 14 - A Sol^{n}

AEM I
F 2007
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(1) $\underline{\nabla} \times \underline{\nabla} f = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \partial f/\partial x & \partial f/\partial y & \partial f/\partial z \end{vmatrix}$

$= \underline{i}\left(\frac{\partial^2 f}{\partial y \partial z} - \frac{\partial^2 f}{\partial z \partial y} \right) - \underline{j}\left(\frac{\partial^2 f}{\partial x \partial z} - \frac{\partial^2 f}{\partial z \partial x} \right) +$

$+ \underline{k}\left(\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial^2 f}{\partial y \partial x} \right) = \underline{0}$ ✓

(2) $\frac{\partial \phi}{\partial x} = e^y$

int. w.r.t. $x \rightarrow \quad \phi = e^y x + g(y,z) \quad (*)$

$\frac{\partial \phi}{\partial y} = x e^y \quad \longleftarrow$ sub in $(*) \Rightarrow$

$e^y x + \frac{\partial g}{\partial y}(y,z) = x e^y \rightarrow \frac{\partial g}{\partial y}(y,z) = 0 \rightarrow$

$g(y,z) = h(z)$. So $(*) \rightarrow$

$\phi = e^y x + h(z) \quad (\dagger)$

$\frac{\partial \phi}{\partial z} = (z+1)e^z \quad \longleftarrow$ sub in $(\dagger) \Rightarrow$

$h'(z) = (z+1)e^z \longrightarrow h(z) = z e^z + K$.

$(\dagger) \rightarrow \quad \underline{\phi = e^y x + z e^z + K}$.

(b) $\phi(1,1,1) - \phi(0,0,0) = e + e + K - K = \underline{2e}$.

(3). $\nabla \times \underline{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3z^2 & \cos y & 2xz \end{vmatrix}$

$= \underline{i}\,(0) - \underline{j}\,(2z - 6z) + \underline{k}\,(0) \neq \underline{0}$

$\underline{F}$ is not conservative.

(4). $\nabla \cdot \nabla \phi = \nabla \cdot \left(\frac{\partial \phi}{\partial x} \underline{i} + \frac{\partial \phi}{\partial y} \underline{j} + \frac{\partial \phi}{\partial z} \underline{k} \right)$

$= \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}$

(5). $\text{div}\,(\text{curl}\ \underline{F}) = \text{div} \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f & g & h \end{vmatrix} =$

$\text{div}\left[\underline{i}\left(\frac{\partial h}{\partial y} - \frac{\partial g}{\partial z} \right) - \underline{j}\left(\frac{\partial h}{\partial x} - \frac{\partial f}{\partial z} \right) + \underline{k}\left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right) \right]$

$= \cancel{\frac{\partial^2 h}{\partial x \partial y}} - \cancel{\frac{\partial^2 g}{\partial x \partial z}} - \cancel{\frac{\partial^2 h}{\partial y \partial x}} + \cancel{\frac{\partial^2 f}{\partial y \partial z}} + \cancel{\frac{\partial^2 g}{\partial z \partial x}} - \cancel{\frac{\partial^2 f}{\partial z \partial y}}$

$= 0$ ✓

(6) $\text{div}\ \underline{F} = 0 + xz + 0 \neq 0$

So by Problem (5), $\underline{F}$ cannot be expressed as curl $\underline{G}$.