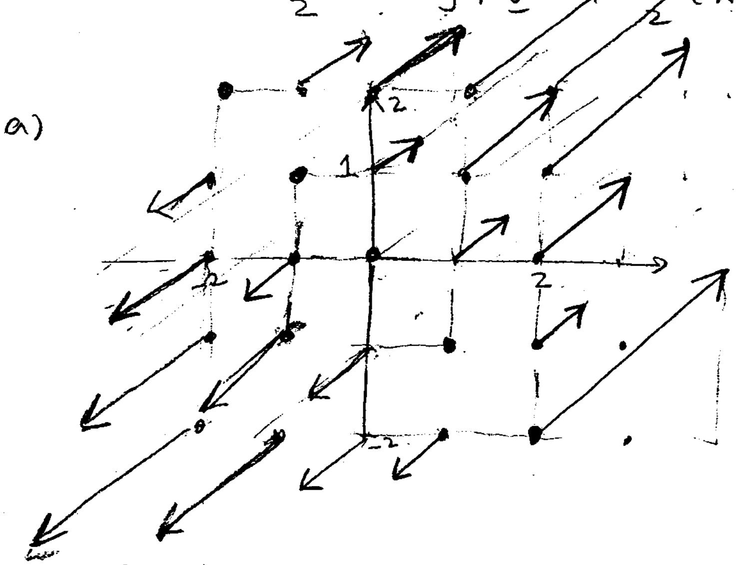
HW 13 - A sol^{n}

AEM I

Fall 2007
Gross

(1.) $\nabla f = \frac{1}{2}(x+y)\,\underline{i} + \frac{1}{2}(x+y)\,\underline{j}$.

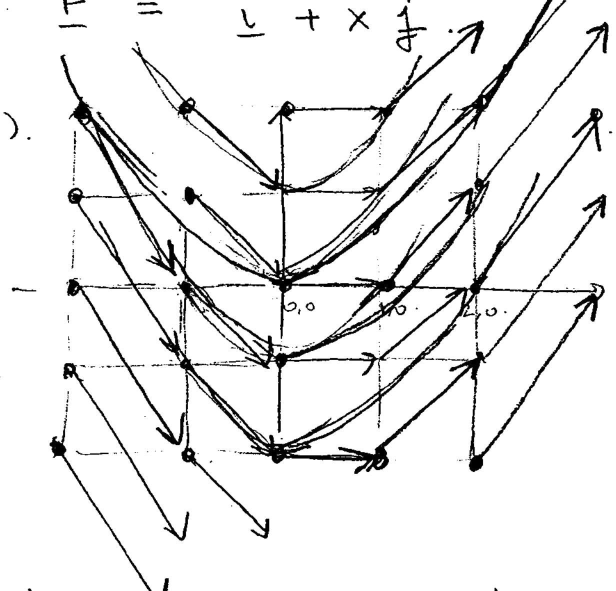
(a)



(b) All nonzero vectors have slope 1. All vectors w/ tails on line $y=-x$ are $\underline{0}$. Vectors point up & right if tails above $y=-x$ & left & down below it.

(2.) $\underline{F} = \underline{i} + x\,\underline{j}$.

(a).



(b) The streamlines look parabolic.

(c) If $\underline{F}$ gives the tangent vectors,

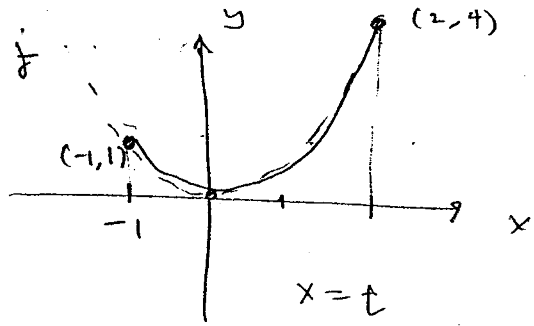
$$\frac{dx}{dt} = 1, \quad \frac{dy}{dt} = x. \quad \Rightarrow \quad \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{x}{1}$$

(d.) $\frac{dy}{dx} = x \rightarrow y = \frac{x^2}{2} + c$.

(0,0) lies on the curve if $c=0$. $\boxed{y = \frac{x^2}{2}}$

(3). $\underline{F}(x,y) = x \sin y \, \underline{i} + y \, \underline{j}$

$\underline{r}(t) = t \, \underline{i} + t^2 \, \underline{j}$



$x = t$

$y = t^2.$

$-1 \le t \le 2.$

$$\int_C \underline{F}(\underline{r}) \cdot d\underline{r}$$

$$= \int_{-1}^{2} \underline{F}(t) \cdot \underline{r}'(t) \, dt$$

$$= \int_{-1}^{2} (t \sin t^2 \, \underline{i} + t^2 \, \underline{j}) \cdot (\underline{i} + 2t \, \underline{j}) \, dt$$

$$= \int_{-1}^{2} [t \sin(t^2) + 2t^3] \, dt = \left[-\frac{\cos(t^2)}{2} + \frac{t^4}{2} \right]$$

$$= -\frac{\cos(4)}{2} + \frac{2^4}{2} + \frac{\cos(1)}{2} - \frac{1}{2} =$$

$$= \boxed{\frac{1}{2} [15 - \cos(4) + \cos(1)].}$$

Note: Is the v.f. conservative? (Does $\underline{F} = \nabla \varphi$ for some φ?)

$\frac{\partial \varphi}{\partial x} = x \sin y$ int. w.r.t. $x \Rightarrow$

$\varphi = \frac{x^2}{2} \sin y + g(y)$ $(*)$

$\frac{\partial \varphi}{\partial y} = y$

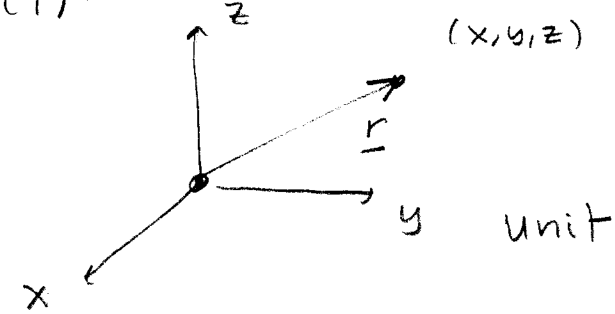
sub in $(*)$. $\frac{x^2}{2} \cos y + g'(y) = y$

contradiction. $g'(y)$ cannot depend on x.

so v.f. not conservative

Alternatively, $\frac{\partial}{\partial y}(x \sin y) \ne \frac{\partial}{\partial x}(y)$ so v.f. not conservative

(4)



distance $= \|\underline{r}\|$

unit direction $= \dfrac{\underline{r}}{\|\underline{r}\|}$

So $\underline{F}(\underline{r}) = \dfrac{k}{\|r\|^2} \dfrac{r}{\|\underline{r}\|}$

$$= \frac{k\,\underline{r}}{\|\underline{r}\|^3} \checkmark$$

Is $\underline{F}$ conservative?

$$\underline{F}(x,y,z) = \frac{k}{(\sqrt{x^2+y^2+z^2})^3}(x\underline{i} + y\underline{j} + z\underline{k})$$

$$\frac{\partial\phi}{\partial x} = \frac{kx}{(\sqrt{x^2+y^2+z^2})^3}$$

Int. w.r.t. x $\to$ $\phi = \dfrac{-k\,\cancel{2}}{(\cancel{2})\sqrt{x^2+y^2+z^2}} + g(y,z)$

$$\frac{\partial\phi}{\partial y} = \frac{ky}{(\sqrt{x^2+y^2+z^2})^3}$$

$\downarrow$

$$-\frac{1}{\cancel{2}}\frac{-\cancel{2}yk}{(\sqrt{x^2+y^2+z^2})^3} + \frac{\partial g}{\partial y} = \frac{ky}{(\sqrt{x^2+y^2+z^2})^3} \to \frac{\partial g}{\partial y} = 0 \to$$

$g(y,z) = h(z)$.

$$\frac{\partial\phi}{\partial z} = \frac{kz}{(\sqrt{x^2+y^2+z^2})^3}$$

$$\frac{zk}{(\sqrt{x^2+y^2+z^2})^3} + h'(z) = \frac{kz}{(\sqrt{x^2+y^2+z^2})^3} \to h'(z) = 0 \to$$

$h(z) = c$.

HW 12.4

$\underline{F} = \nabla \phi \ , \quad \phi = \dfrac{-k}{\sqrt{x^2+y^2+z^2}} + c.$

$$\int_C \underline{F} \cdot d\underline{r} = \phi(2,1,5) - \phi(2,0,0).$$

$$= \boxed{k\left(\frac{1}{2} - \frac{1}{\sqrt{30}}\right)}$$