AcmI
F2007
Gross

HW1 Sol^N

(1). (a). $\dfrac{7+3i}{4+4i-1} = \dfrac{(7+3i)(3-4i)}{(3+4i)(3-4i)} = \dfrac{21-19i+12}{9+16}$

$= \boxed{\dfrac{33}{25} - \dfrac{19}{25}i}$ ✓

(b). $\dfrac{4+4i-1}{(-1+8i)^2} = \dfrac{3+4i}{1-16i-64} = \dfrac{(3+4i)\ (-63+16i)}{(-63-16i)(-63+16i)} =$

$= \dfrac{-189+48i-252i-64}{3969+256} = \boxed{\dfrac{-253}{4225} - \dfrac{204}{4225}i}$ ✓

(c) $(2+i)(-1-i)(3-2i) = (-2-3i+1)(3-2i) =$

$= (-1-3i)(3-2i) = -(1+3i)(3-2i) = -(3+7i+6)$

$= \boxed{-9-7i}$ ✓

(2) $(-1+i)^3 - 2(-1+i) - 4 \overset{?}{=} 0$ (similarly for $-1-i$)

$(1+1-2i-1)(-1+i) + 2-2i-4 = -2i(-1+i)-2-2i =$

$= 2i+2-2-2i = 0$ ✓

(3) $(x+iy)^3 = 1 \iff (x^2+2ixy-y^2)(x+iy) = 1 \iff$

$\underline{x^3} + 2ix^2y - \underline{xy^2} + ix^2y - \underline{2xy^2} - iy^3 = 1 \iff$

$\boxed{x^3 - 3xy^2 = 1 \quad \& \quad 3x^2y - y^3 = 0.}$

Solving these two eq'ns simultaneously for x&y is equivalent to solving $z^3=1$ for z, where $z = x+iy$.

(4) $z^3 + v^2 = 4i$ $\qquad$ $\left.\begin{array}{l} z = x+iy \\ v = u+iw \end{array}\right\} \rightarrow$

$v - 2i + 3 = 0$

$(x+iy)^3 + (u+iw)^2 = 4i$, $\quad u+iw = 2i-3$. $\rightarrow$

$x^3 + 3ix^2y - 3xy^2 - iy^3 + u^2 + 2iuw - w^2 = 4i$,

$u+iw = 2i-3$ $\rightarrow$

$$\left.\begin{array}{ll} x^3 - 3xy^2 + u^2 - w^2 = 0, & u = -3 \\ 3x^2y - y^3 + 2uw = 4, & w = 2 \end{array}\right\}$$

Solving these 4 real eq'ns simultaneously for x, y, u, w

is equivalent to solving the 2 cpx eq'ns (*) simultaneously for v & z ($v = u+iw$, $z = x+iy$).

(5) $\mathrm{Re}(iz) = \mathrm{Re}(i(x+iy)) = \mathrm{Re}(-y+ix) = -y$

$-\mathrm{Im}(z) = -\mathrm{Im}(x+iy) = -y$ $\leftarrow$ $=$ ✓.

(6) $a<b$ only makes sense for numbers you can place on a real-number line. You can compare the real or imaginary parts of z_1 & z_2, e.g. $\mathrm{Re}\, z_1 < \mathrm{Re}\, z_2$, but not the complex numbers themselves.