Consider a vector space S over $\mathbb{R}$ with addition and scalar multiplication defined.

- **Definition:** A **linear combination** of elements $a_1, a_2, \ldots, a_n \in S$ has the form
$$\alpha_1 a_1 + \alpha_2 a_2 + \cdots + \alpha_n a_n,$$
where $\alpha_i \in \mathbb{R}$, $i = 1 \ldots n$.

  Notice that the linear combination is in S because S is closed under addition and scalar mulitiplication.

- **Definition:** The **span** of a set of elements $a_1, a_2, \ldots, a_n \in S$ consists of *all* linear combinations of $a_1, a_2, \ldots, a_n$. The set is denoted $\text{Span}(a_1, a_2, \ldots, a_n)$.

- **Definition:** The set of elements $a_1, a_2, \ldots, a_n \in S$ **spans the vector space** S if *every* element in S can be expressed as a linear combination of $a_1, a_2, \ldots, a_n$.

- **Definition:** A set of elements in S is **linearly dependent** if one of the elements can be expressed as a linear combination of the others.

- **Definition:** A set of elements $a_1, a_2, \ldots, a_n \in S$ is **linearly independent** if the only way to satisfy the equation
$$\alpha_1 a_1 + \alpha_2 a_2 + \cdots + \alpha_n a_n = 0$$
is by picking *every* value of α_i to be 0.

- **Theorem:** A set of elements in S is either linearly independent or linearly dependent and not both.

- **Definition:** A set of elements in S is a **basis** of S if it is linearly independent and spans S.

- **Definition:** The **dimension** of the vector space S is the number of elements in a basis of S.